CSE 332
Introduction to Visualization

High-Dimensional Data

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Lecture	Topic	Projects
1	Intro, schedule, and logistics	
2	Applications of visual analytics, data, and basic tasks	
3	Data preparation and reduction	Project 1 out
4	Data preparation and reduction	
5	Data reduction and similarity metrics	
6	Dimension reduction	
7	Introduction to D3	Project 2 out
8	Bias in visualization	
9	Perception and cognition	
10	Visual design and aesthetics	
11	Cluster and pattern analysis	
12	High-Dimensional data visualization: linear methods	
13	High-D data vis.: non-linear methods, categorical data	Project 3 out
14	Principles of interaction	
15	Visual analytics and the visual sense making process	
16	VA design and evaluation	
17	Visualization of graphs and hierarchies	
18	Visualization of time-varying and time-series data	Project 4 out
19	Midterm	
20	Maps and geo-vis	
21	Computer graphics and volume rendering	
22	Techniques to visualize spatial (3D) data	Project 4 halfway report due
23	Scientific and medical visualization	
24	Scientific and medical visualization	
25	Non-photorealistic rendering	
26	Memorable visualizations, visual embellishments	Project 5 out
27	Infographics design	
28	Projects Hall of Fame demos	

Understanding High-D Objects

Feature vectors are typically high dimensional

- this means, they have many elements
- high dimensional space is tricky
- most people do not understand it
- why is that?

- well, because you don't learn to see high-D when your vision system develops

Object permanence (Jean Piaget)

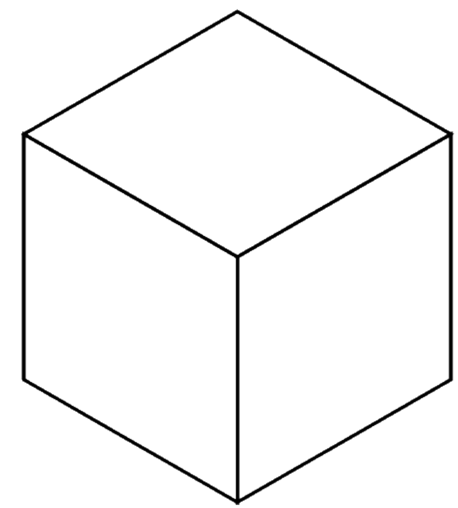
- the ability to create mental pictures or remember objects and people you have previously seen
- thought to be a vital precursor to creativity and abstract thinking

High-D Space is Tricky

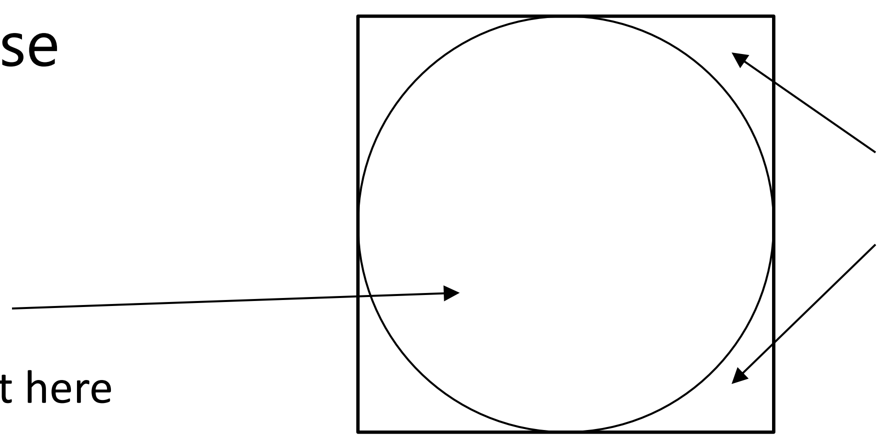
The curse of dimensionality

As *n (number of dimensions)* $\to \infty$

- Cube: side length l, diagonal d, volume V
- $V \to \infty$ for $l > 1$
- $V \to 0$ for $l < 1$
- $V = 1$ for $l = 1$
- $d \to \infty$

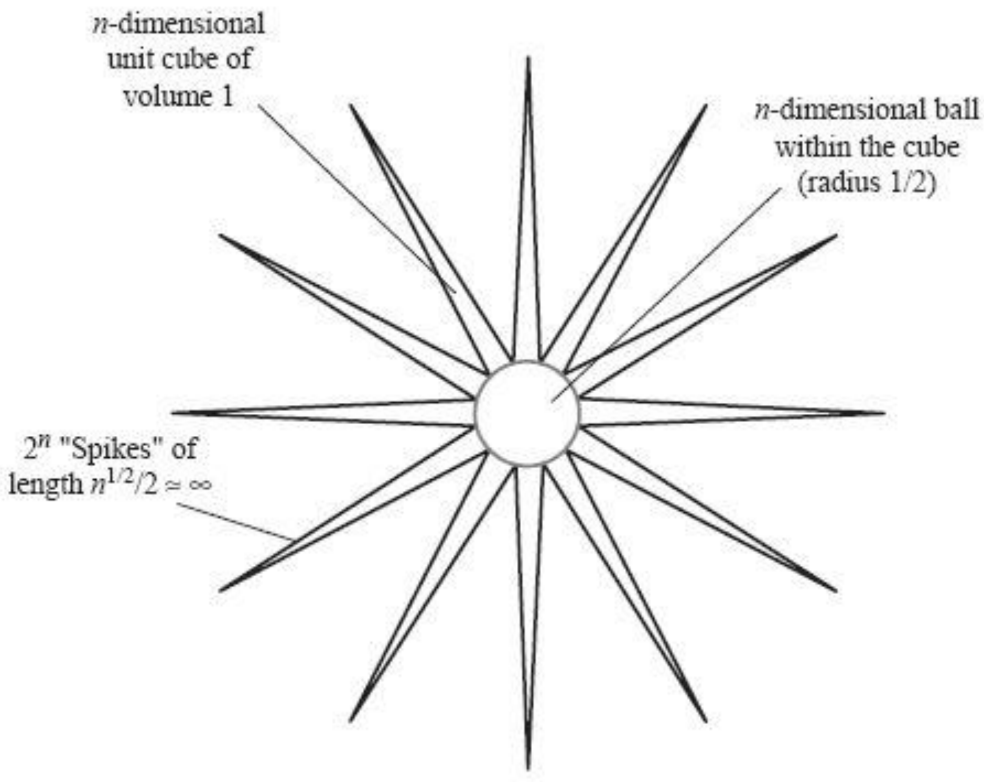


and very sparse



High-D Space is Tricky

Essentially hypercube is like a “hedgehog”



Curse of Dimensionality

Points are all at about the same distance from one another

- concentration of distances
- fundamental equation (Bellman, '61)

$$\lim_{n \to \infty} \frac{Dist_{\max} - Dist_{\min}}{Dist_{\min}} \to 0$$

- so as *n* increases, it is impossible to distinguish two points by (Euclidian) distance
  - unless these points are in the same cluster of points

Sparseness Demonstration

Space gets extremely sparse

- with every extra dimension points get pulled apart further
- distances become meaningless

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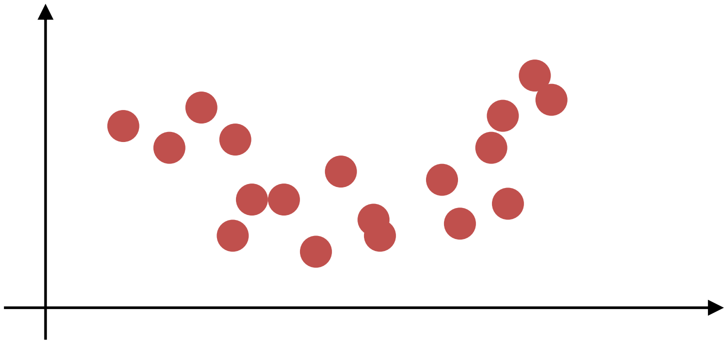
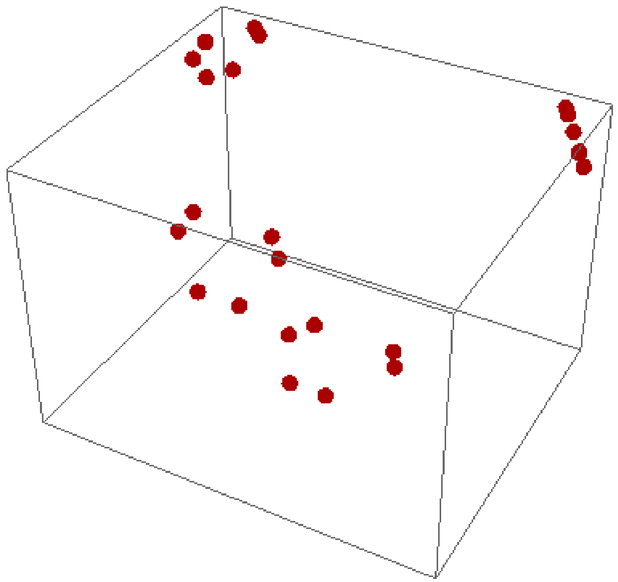
1D – points are very close



2D – points spread apart



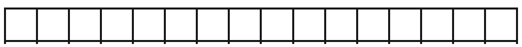
3D – getting even sparser

4D, 5D, … – sparseness grows further

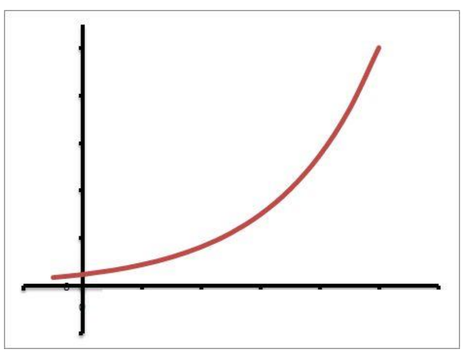
Space and Memory Management

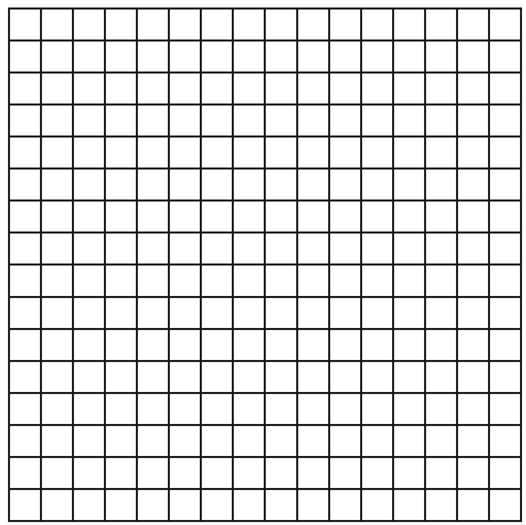
Indexing (and storage) also gets very expensive

- exponential growth in the number of dimensions

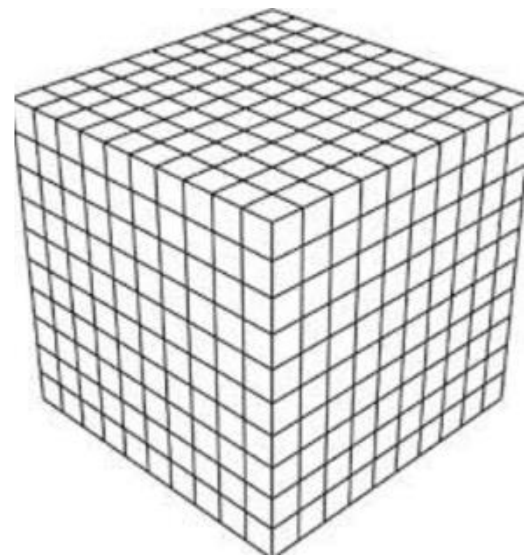


16 cells





16^2 = 256 cells



16^3 = 4,096 cells

- 4D: 65k cells   5D: 1M cells   6D: 16M cells   7D: 268M cells
- keep a keen eye on storage complexity

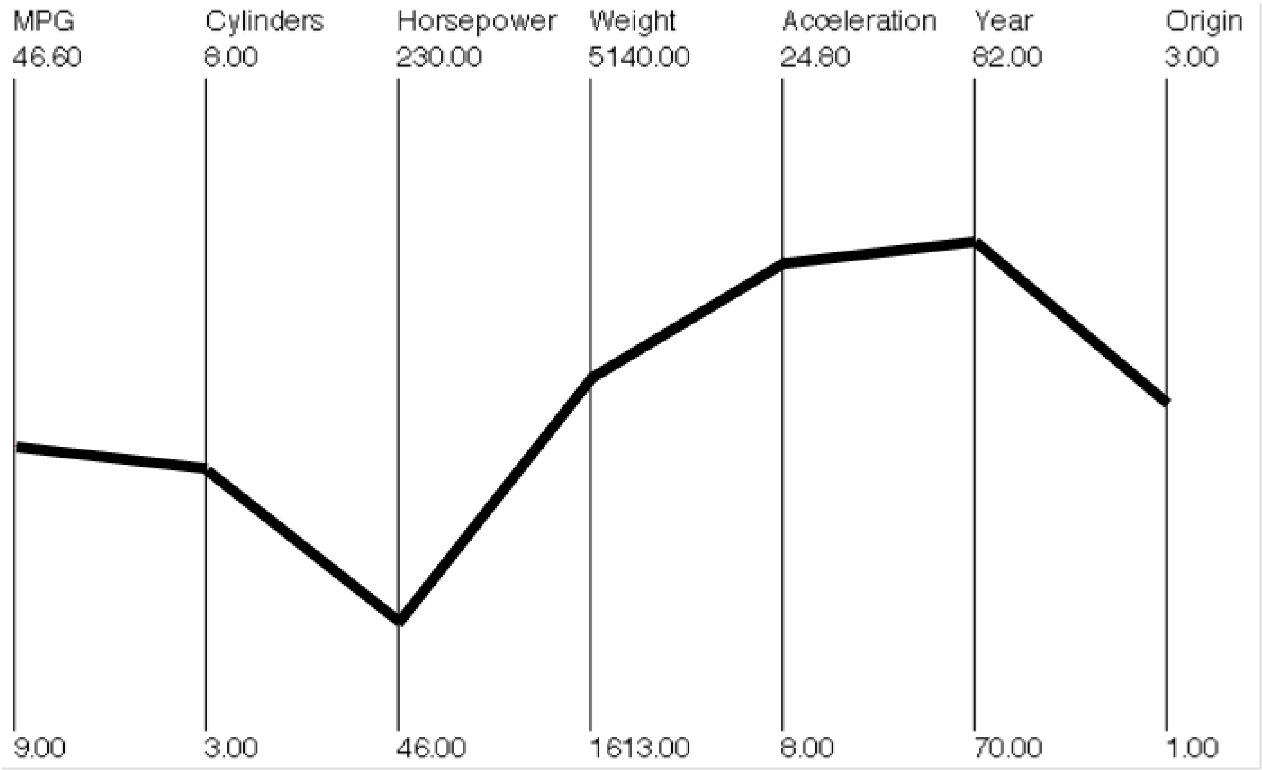
Parallel Coordinates

Invented by Al Inselberg in the early 1990s

Good way to see raw high-dimensional data

- but there are shortcomings
- we will see

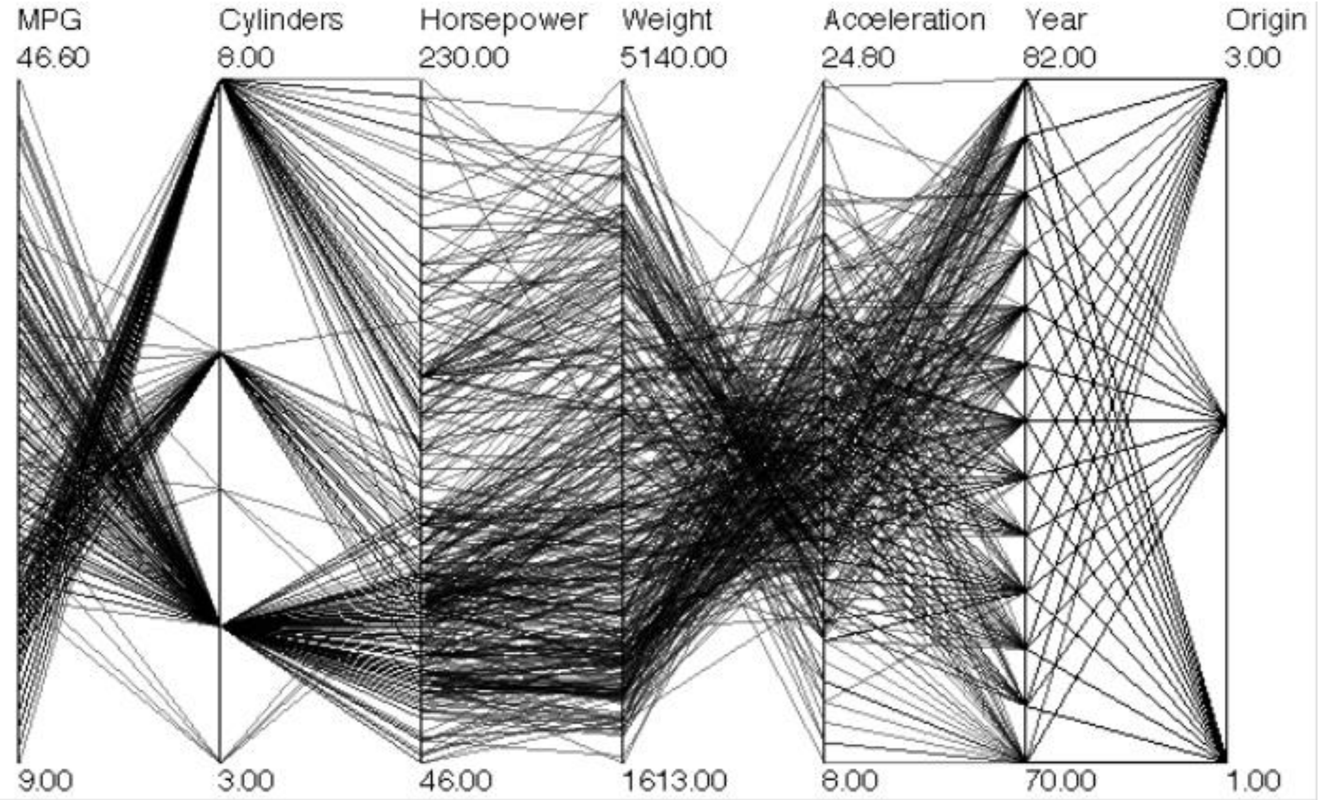
Parallel Coordinates – 1 Car



The N=7 data axes are arranged side by side

- in parallel

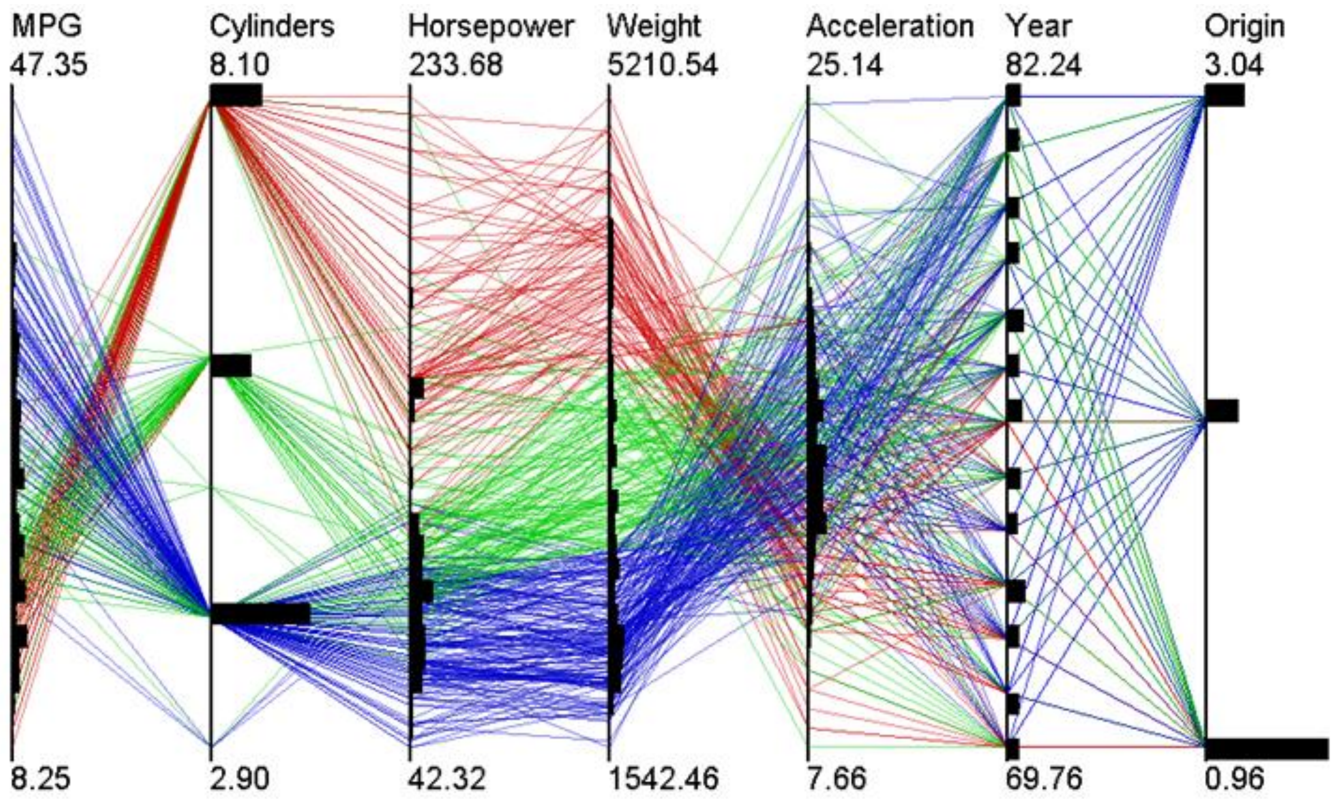
Parallel Coordinates – 100 Cars



Hard to see the individual cars?

- what can we do?

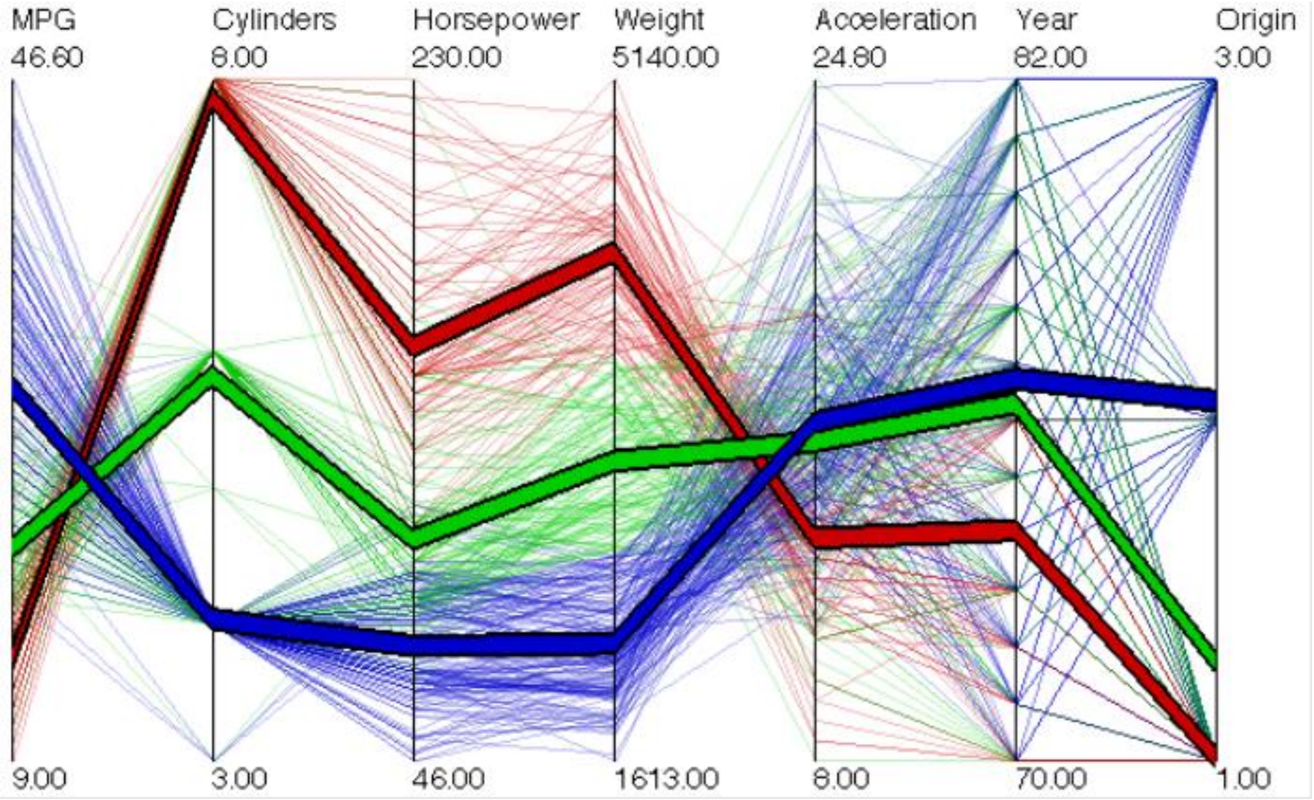
Parallel Coordinates – 100 Cars



Grouping the cars into sub-populations

- we perform clustering
- an be automated or interactive (put the user in charge)

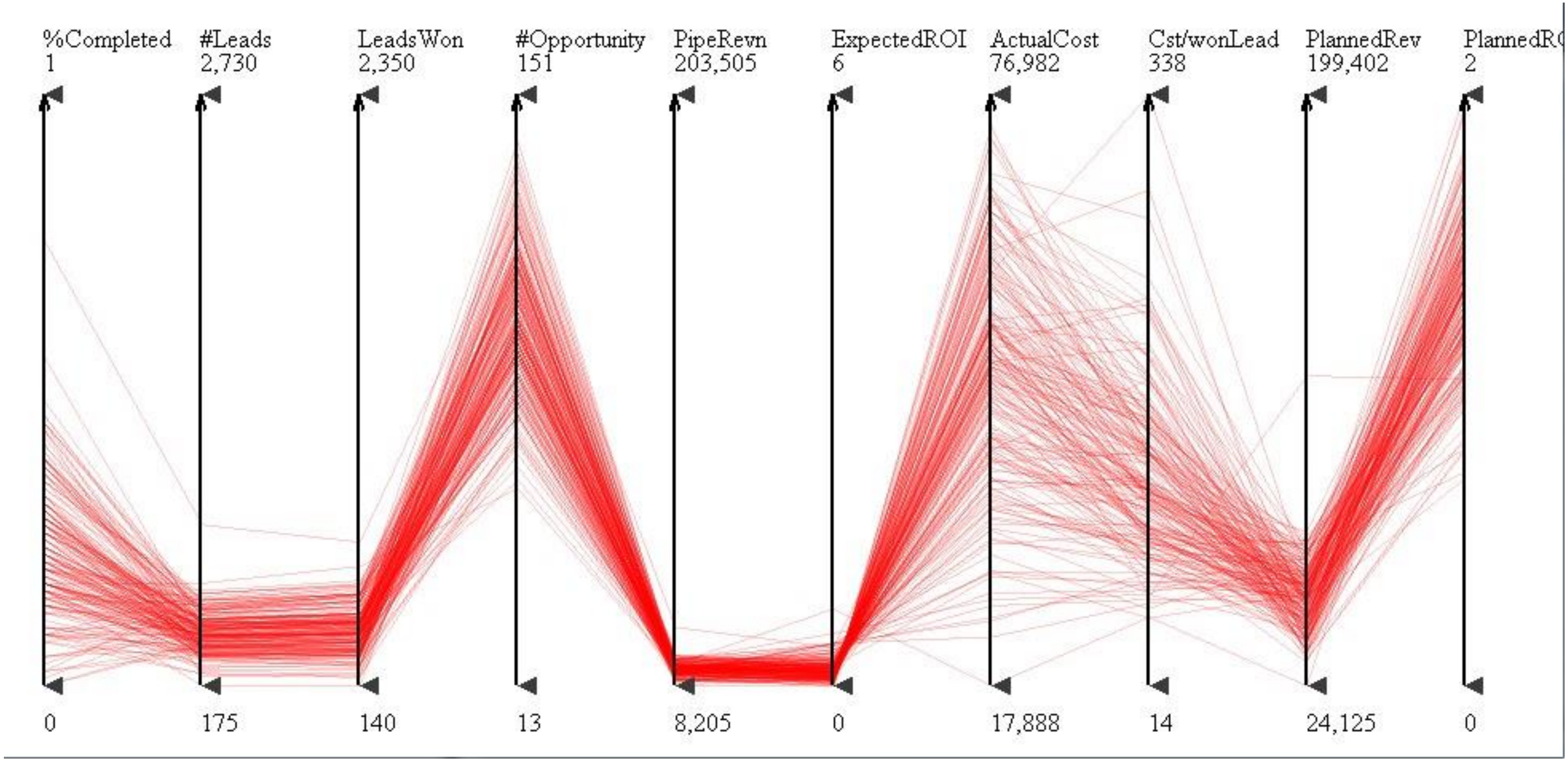
PC With Mean Trend



Computes the mean and superimposes it onto the lines

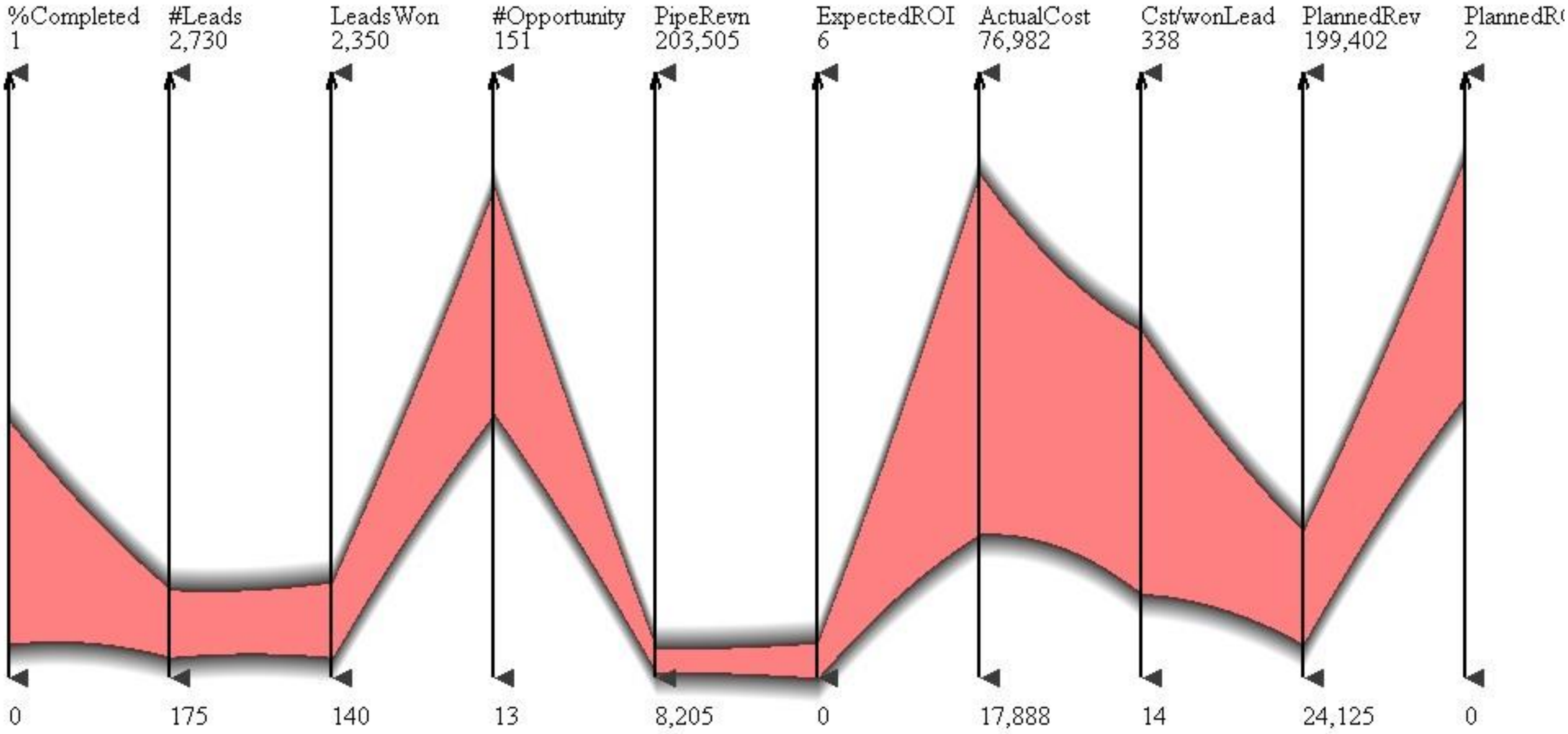
- allows one to see trends

PC With Illustrative Abstraction



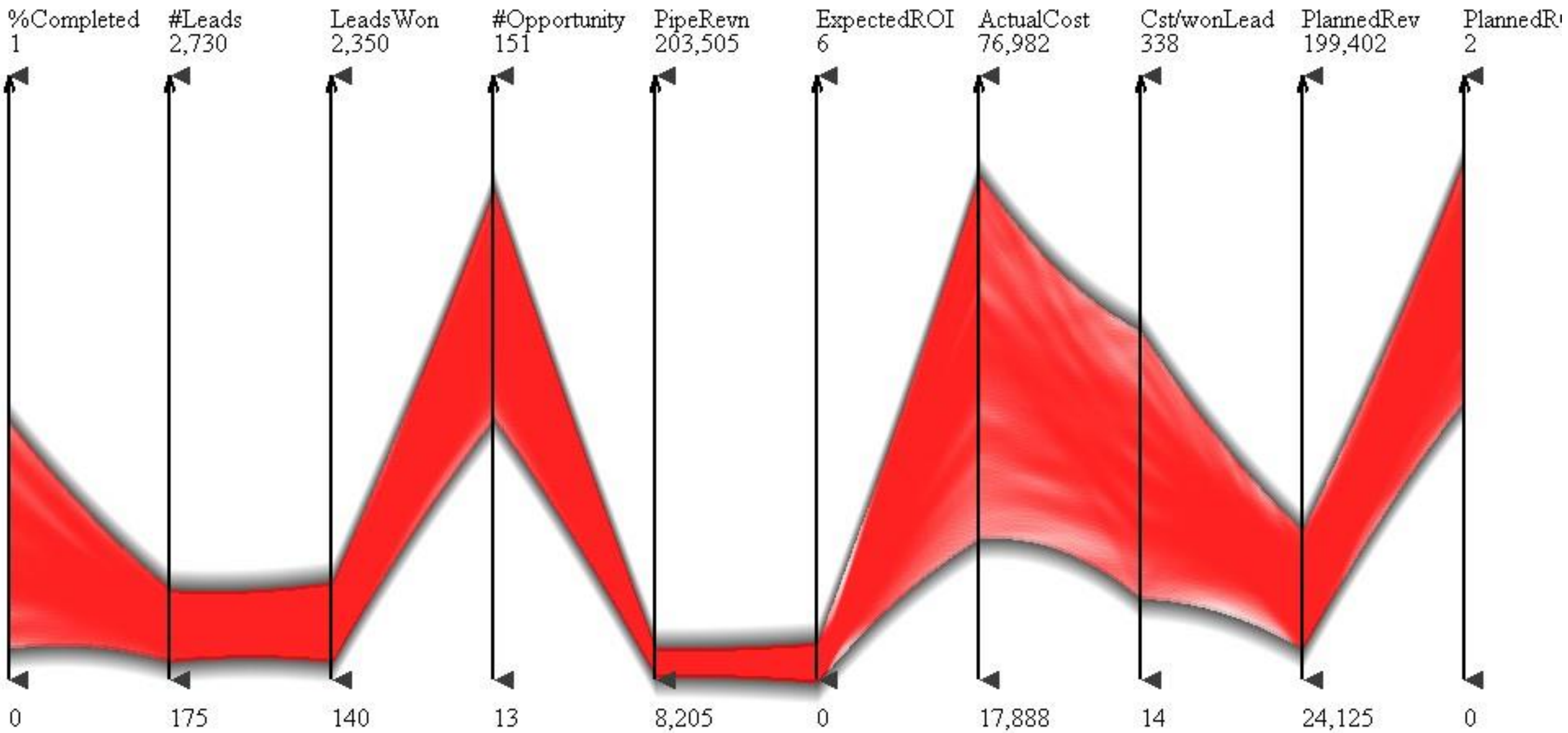
individual polylines

PC With Illustrative Abstraction



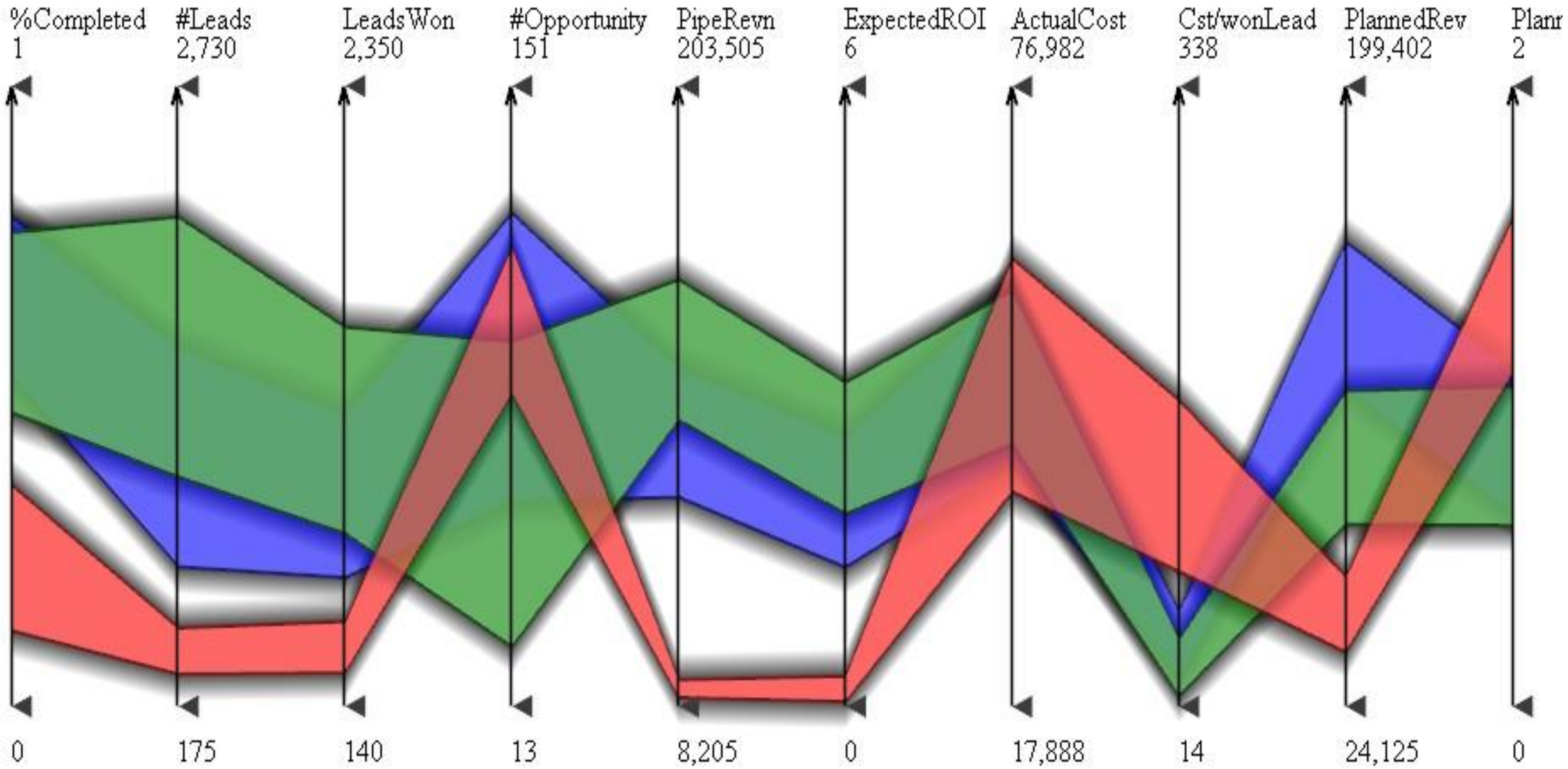
completely abstracted away

PC With Illustrative Abstraction



blended partially

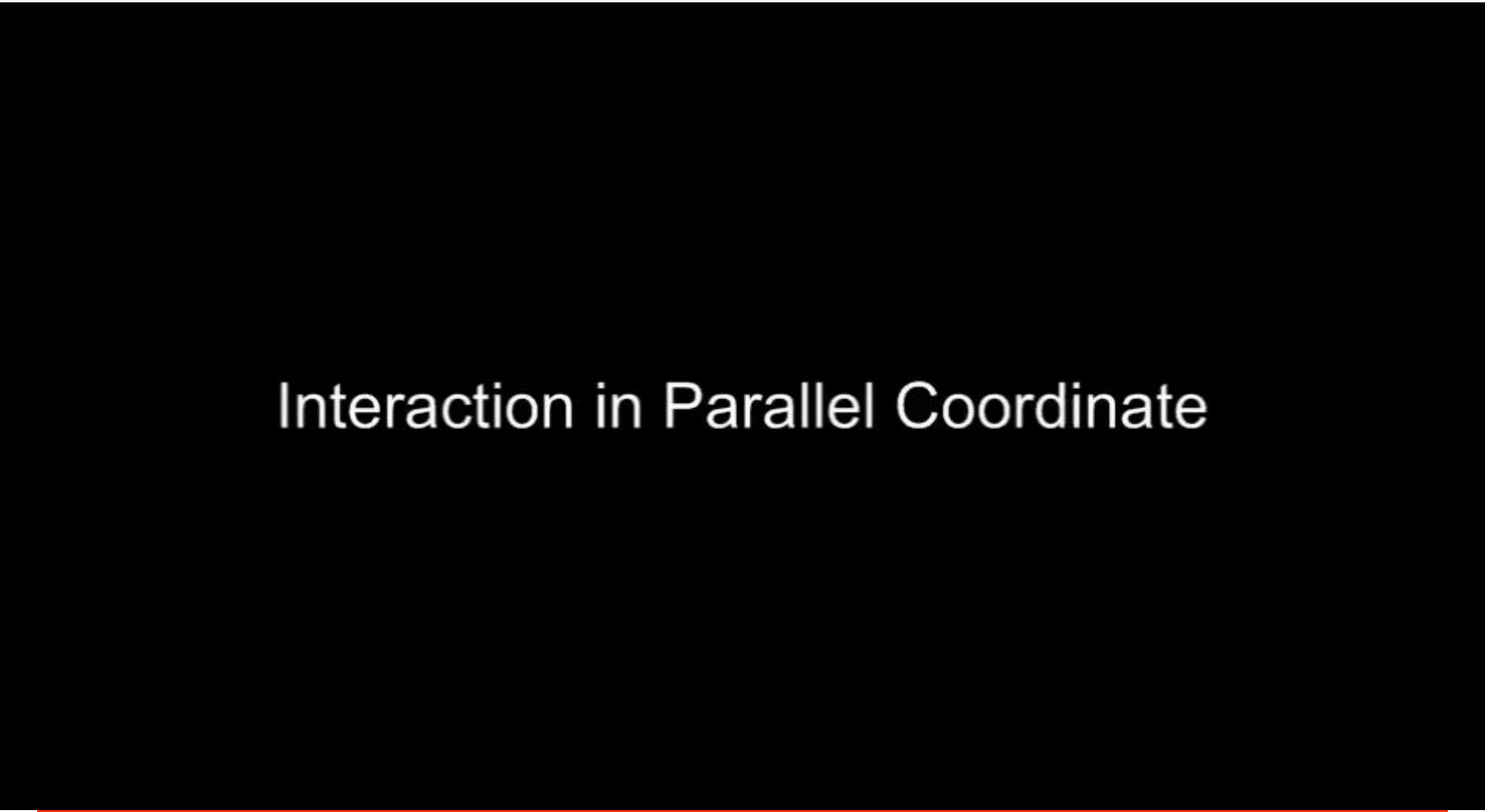
PC With Illustrative Abstraction



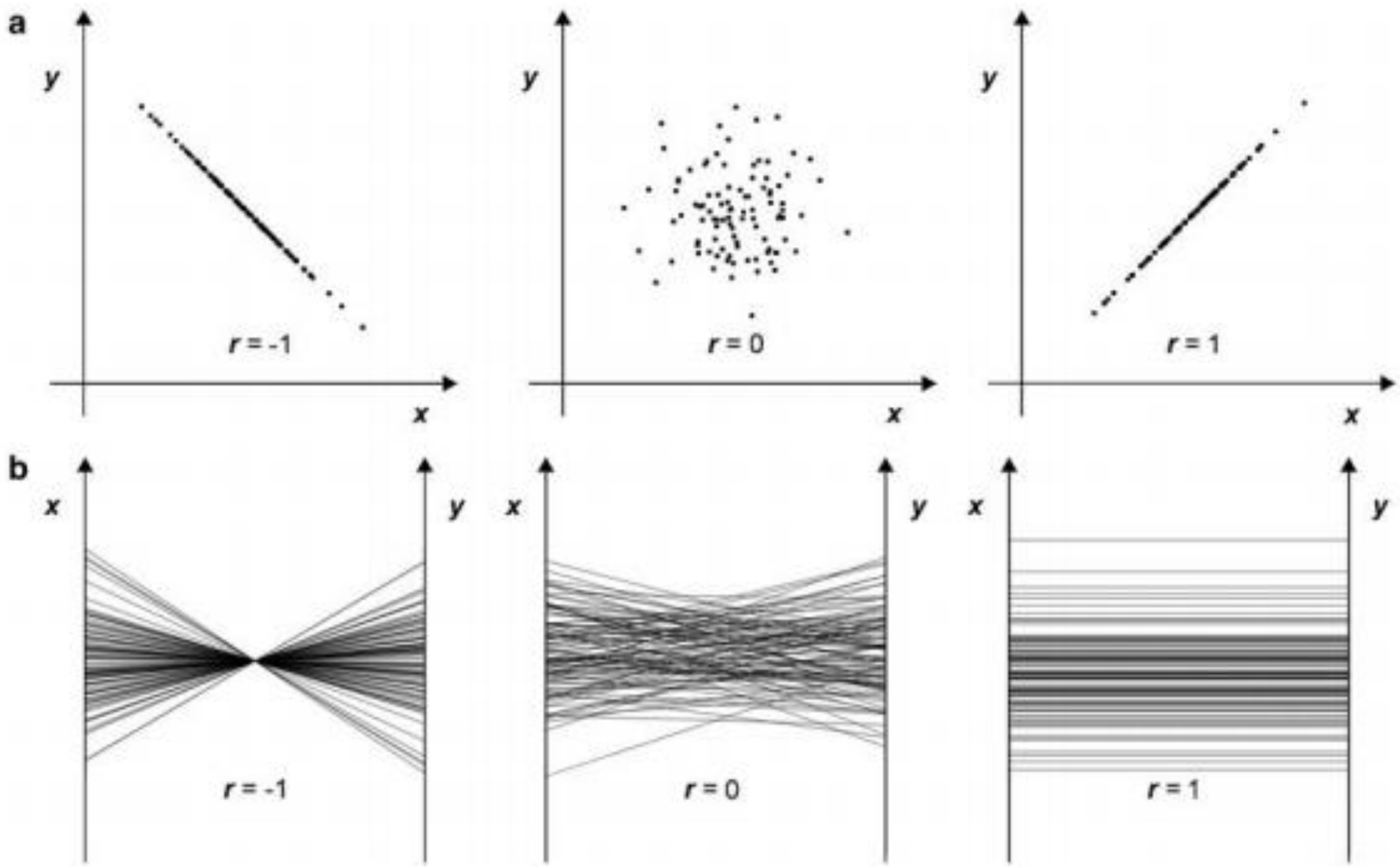
all put together – three clusters

[McDonnell and Mueller, 2008]

Interaction is Key



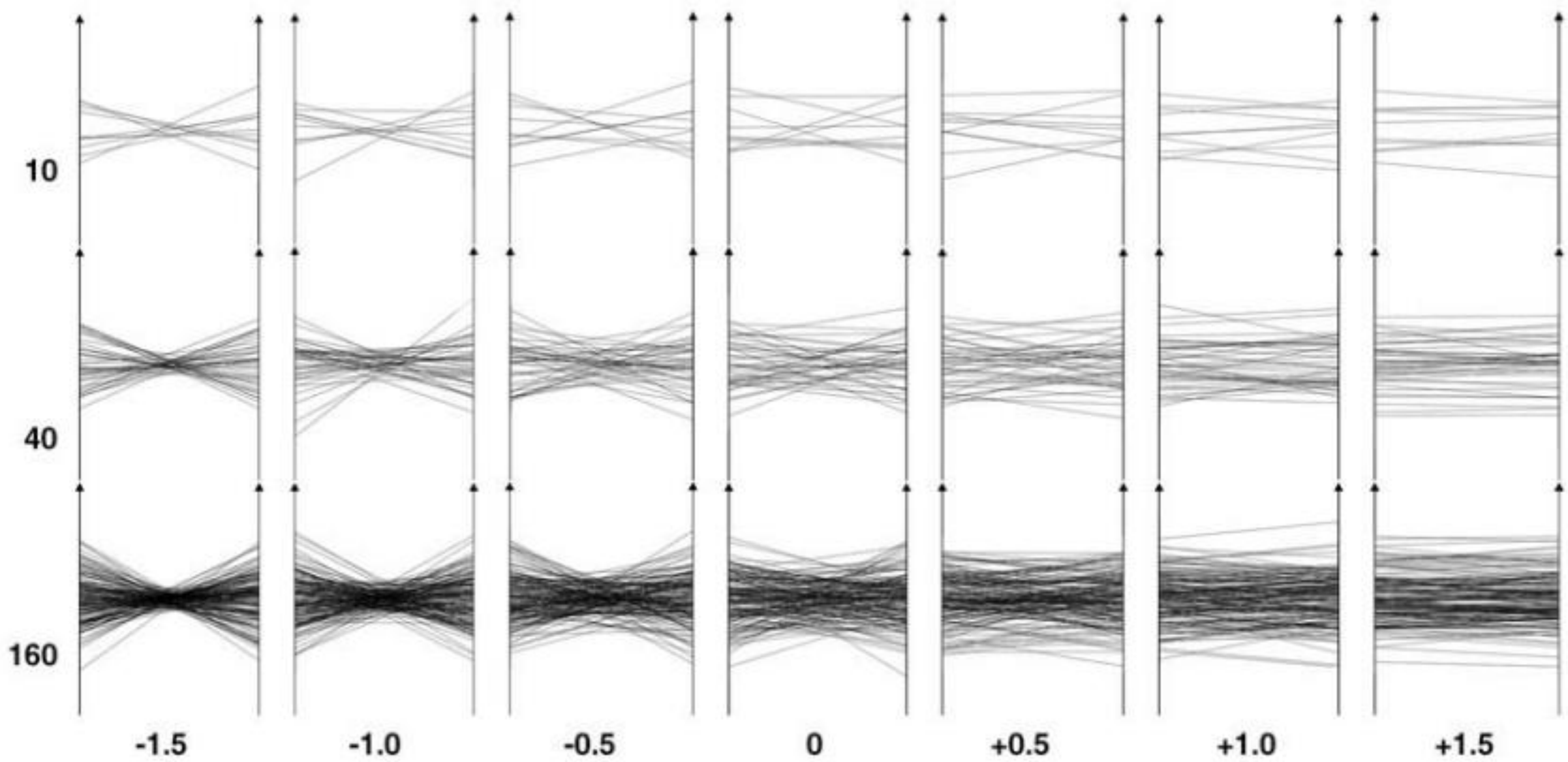
Patterns in Parallel Coordinates



correlation r=-1.0 r=0 r=1.0

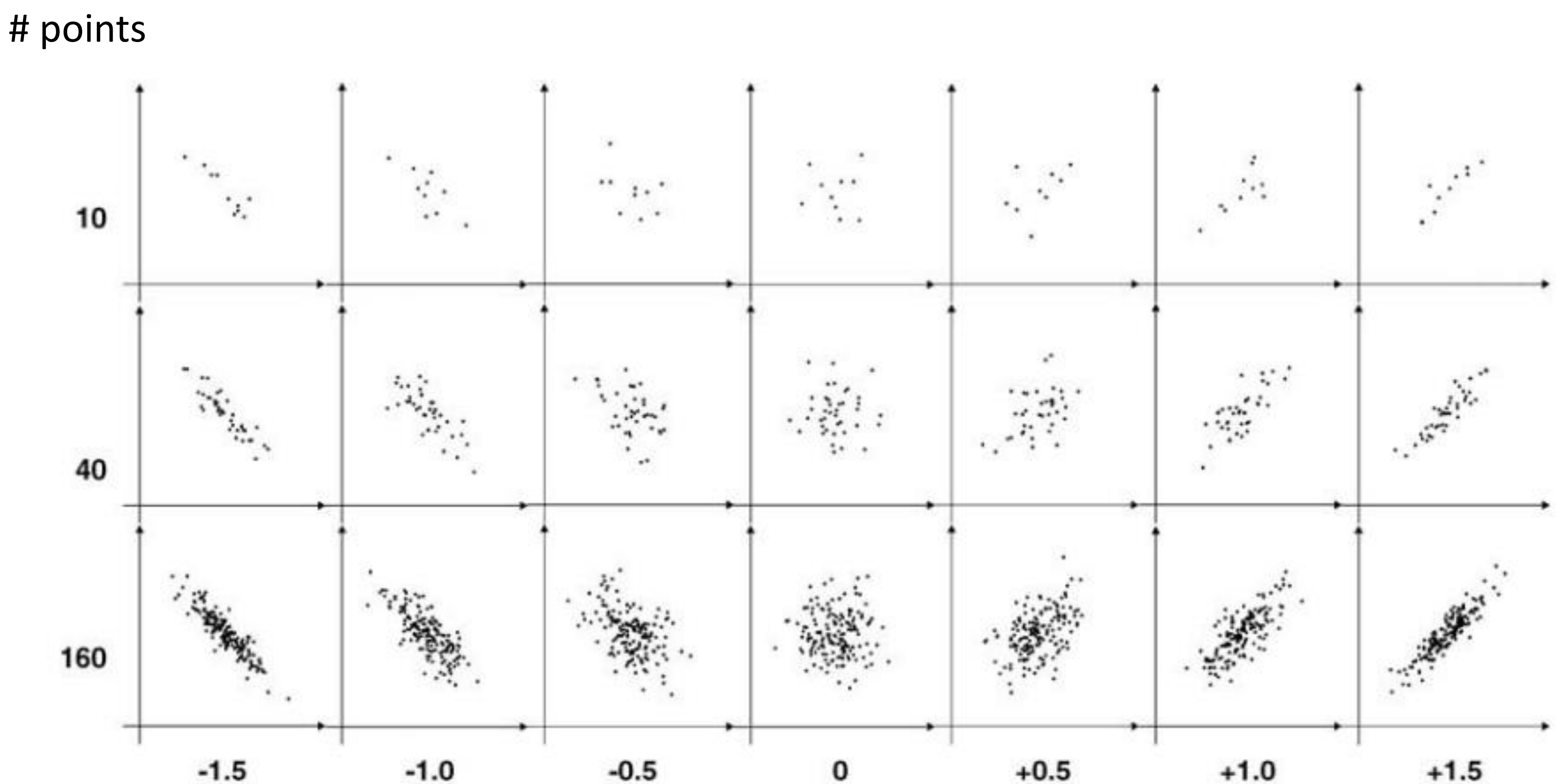
Patterns in Parallel Coordinates

points



Fisher-z (corresponding to $\rho = 0, \pm 0.462, \pm 0.762, \pm 0.905$)

Patterns in Scatterplots



Fisher-z (corresponding to $\rho = 0, \pm 0.462, \pm 0.762, \pm 0.905$)

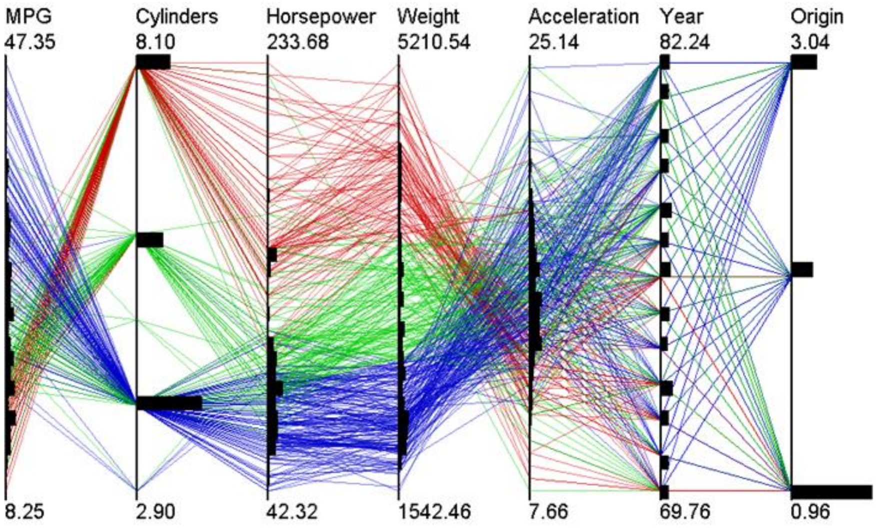
Li et al. found that twice as many correlation levels can be distinguished with scatterplots

Information Visualization Vol. 9, 1, 13 – 30

Axis Reordering Problem

There are n! ways to order the n dimensions

- how many orderings for 7 dimensions?
- 5,040
- but since can see relationships across 3 axes a better estimate is n!/((n-3)! 3!) = 35
- still a lot of axes orderings to try out → we need help

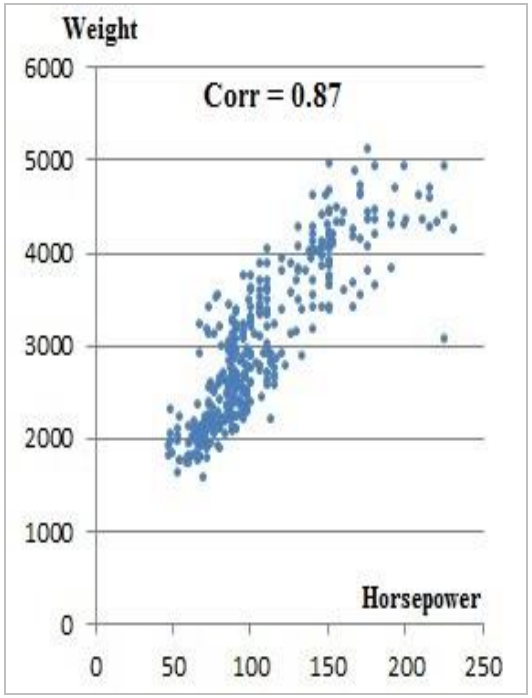


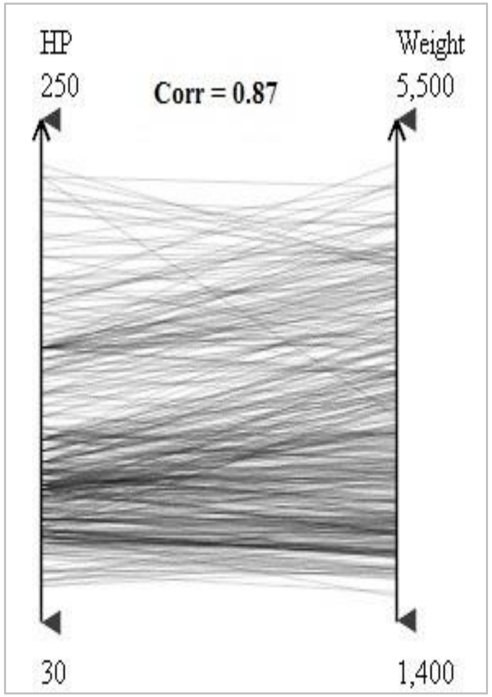
We Need a Measure for Relationships

Correlation

- a statistical measure that indicates the extent to which two or more variables fluctuate together

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$





Building the Correlation Matrix

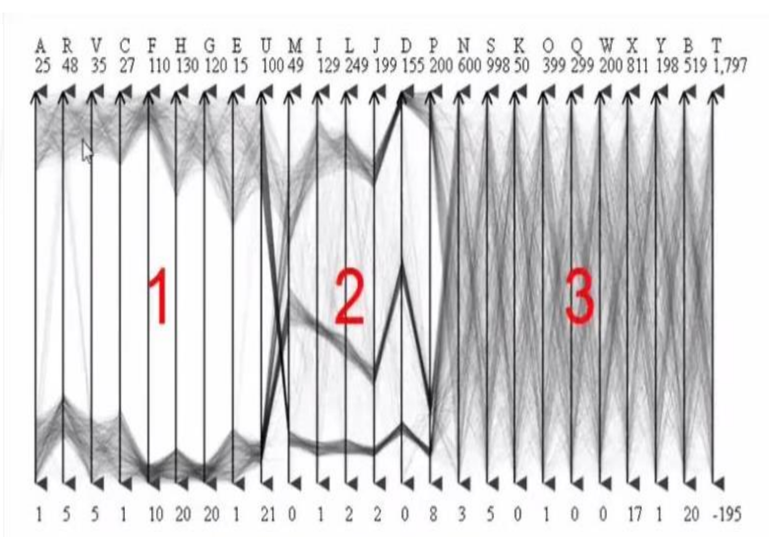
[Zhang and Mueller, 2012]

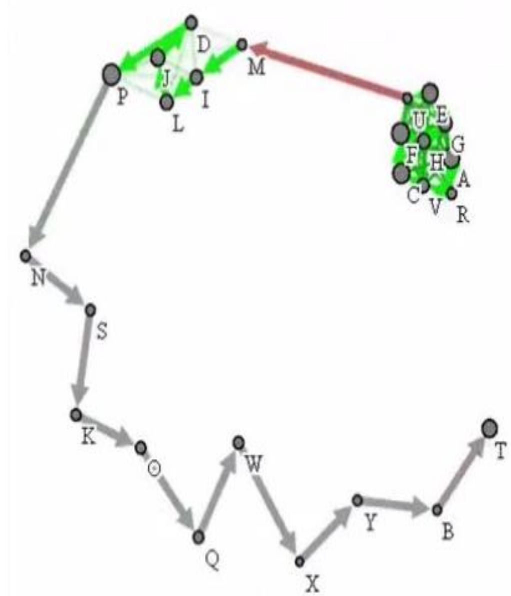
Create a correlation matrix

Run a mass-spring model

Run Traveling Salesman on the correlation nodes

Use it to order your parallel coordinate axes via TSP

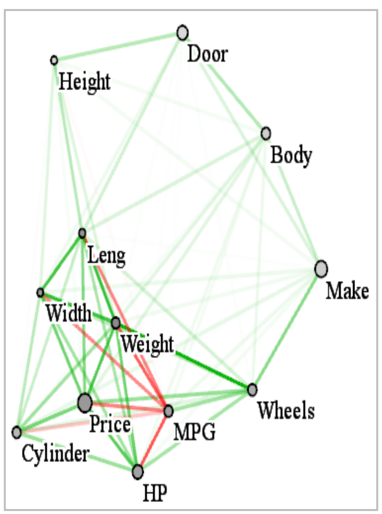




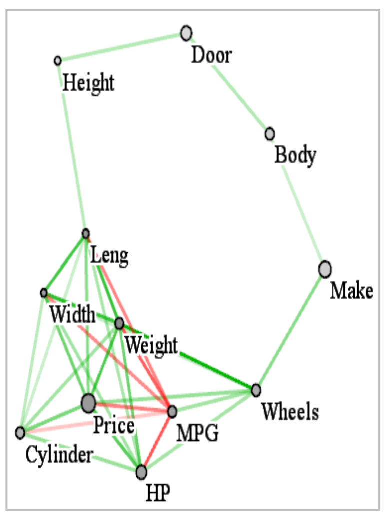
	MRK	MSFT	PFE	PG	*T*	TRV	UTX	VZ	WMT	XOM
MRK	1.	0.39	0.72	−0.43	0.57	0.031	−0.26	0.61	−0.11	−0.25
MSFT	0.39	1.	0.14	0.11	0.56	0.25	0.25	0.67	−0.074	0.24
PFE	0.72	0.14	1.	−0.77	0.08	−0.37	−0.65	0.19	−0.077	−0.72
PG	−0.43	0.11	−0.77	1.	0.25	0.68	0.92	0.086	0.072	0.9
T	0.57	0.56	0.08	0.25	1.	0.65	0.46	0.87	−0.059	0.54
TRV	0.031	0.25	−0.37	0.68	0.65	1.	0.83	0.43	−0.0067	0.81
UTX	−0.26	0.25	−0.65	0.92	0.46	0.83	1.	0.27	−0.033	0.93
VZ	0.61	0.67	0.19	0.086	0.87	0.43	0.27	1.	0.026	0.36
WMT	−0.11	−0.074	−0.077	0.072	−0.059	−0.0067	−0.033	0.026	1.	0.032
XOM	−0.25	0.24	−0.72	0.9	0.54	0.81	0.93	0.36	0.032	1.

Interaction with the Correlation Network

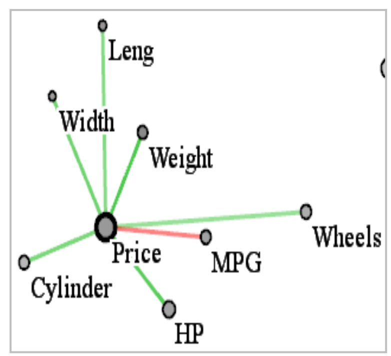
- Vertices are attributes, edges are correlations
  - vertex: size determined by $\sum_{j=0}^{D} \frac{|correlation(i,j)|}{D-1}\ j \neq i$
  - edge: color/intensity → sign/strength of correlation



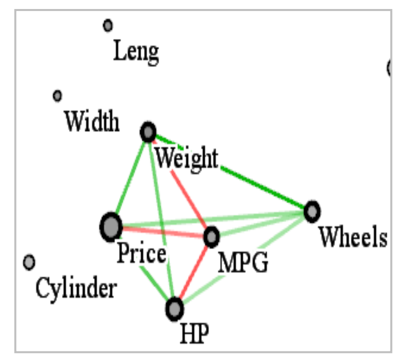
all edges



filtered by strength



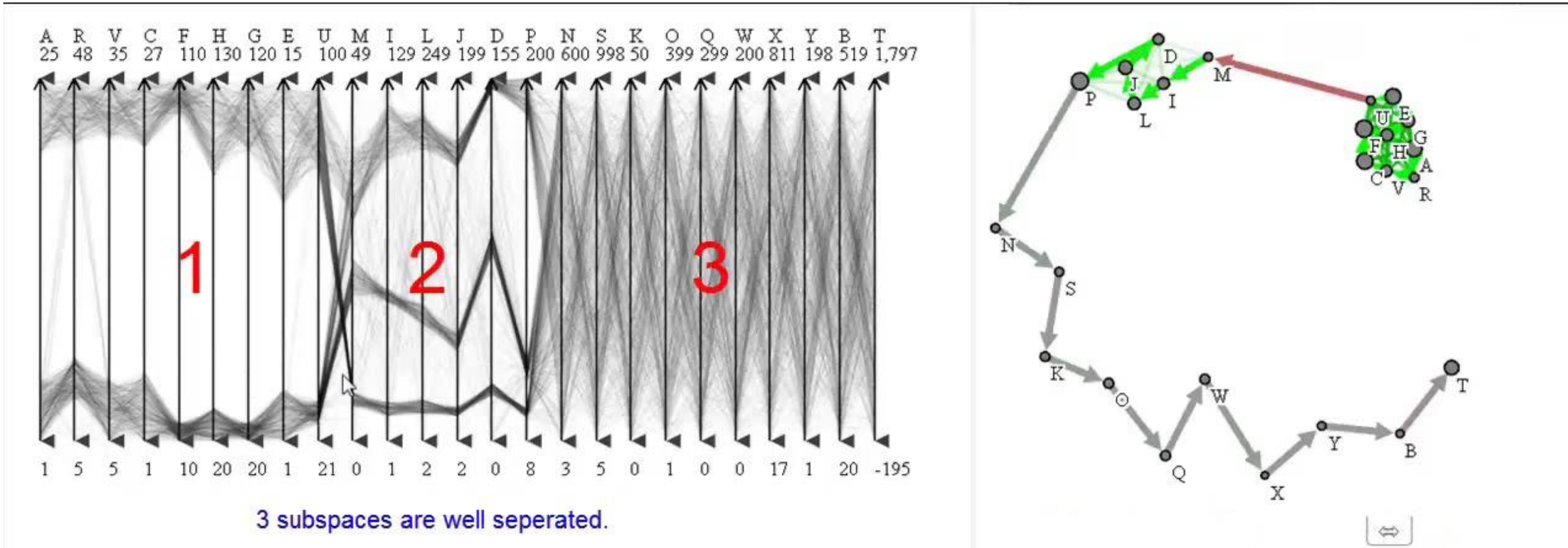
attribute centric



subset of attributes

[Zhang and Mueller, 2014]

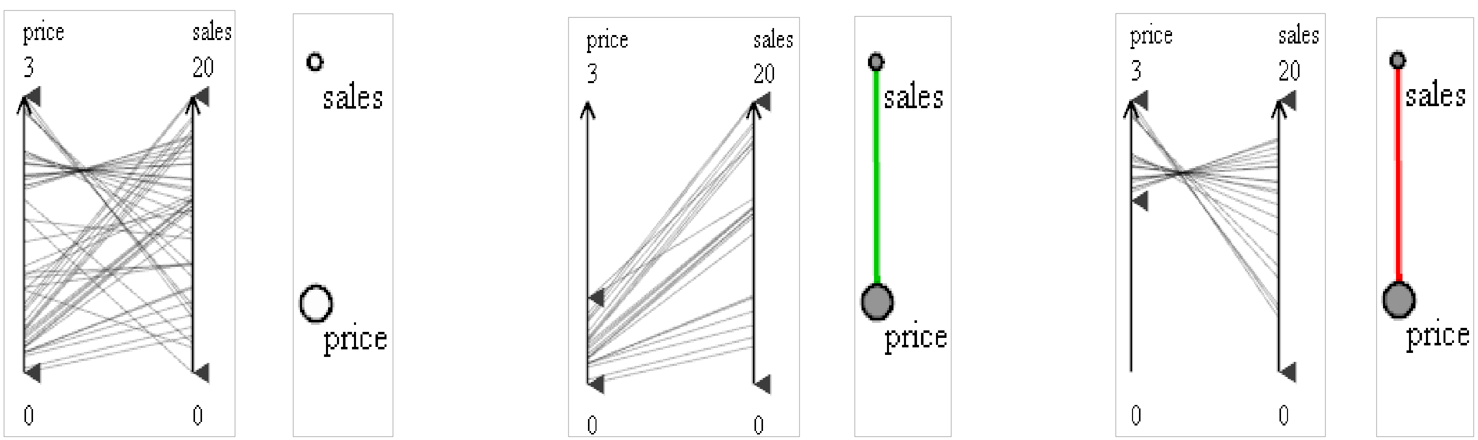
Multiscale Zooming



Bracketing and Conditioning

Correlation strength can often be improved by constraining a variable's value range

- this limits the derived relationships to this value range
- such limits are commonplace in targeted marketing, etc.



no bracketing lower price range higher price range

[Zhang and Mueller, 2014]

Parallel Sets

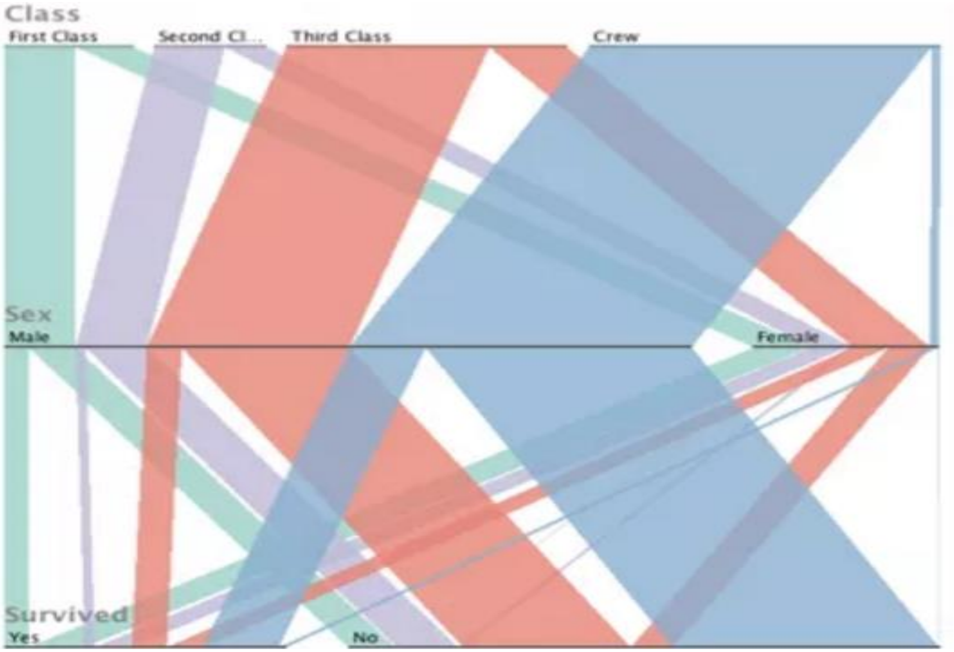
Developed by [Kosara et al. TVCG, 2006]

Parallel coordinates for categorical data

- for example, census and survey data, inventory, etc.
- data that can be summed up in a cross-tabulation

Example

- Titanic dataset
- what can we see here?

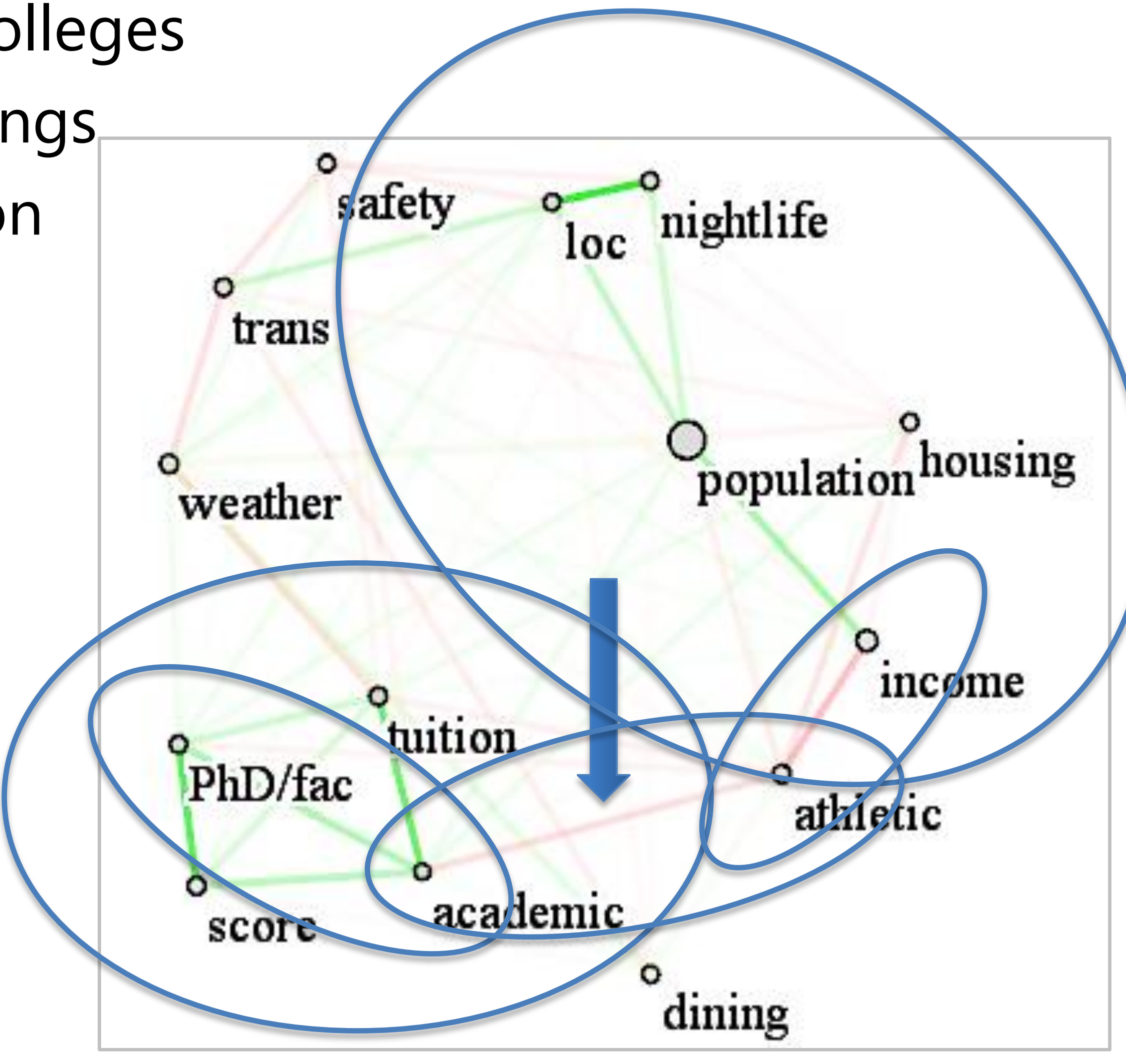


Correlation Plots Are Powerful

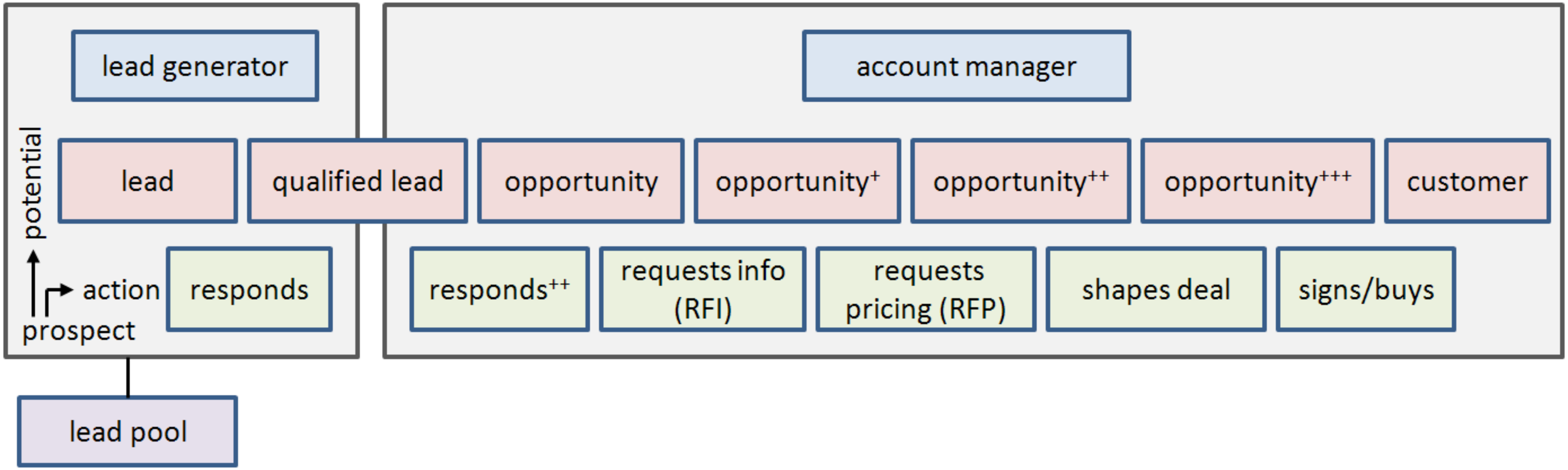
Fused dataset of 50 US colleges

US News: academic rankings

College Prowler: survey on campus life attributes



Anatomy of a Sales Pipeline



The Setup

Scene:

- a meeting of sales executives of a large corporation, Vandelay Industries

Mission:

- review the strategies of their various sales teams

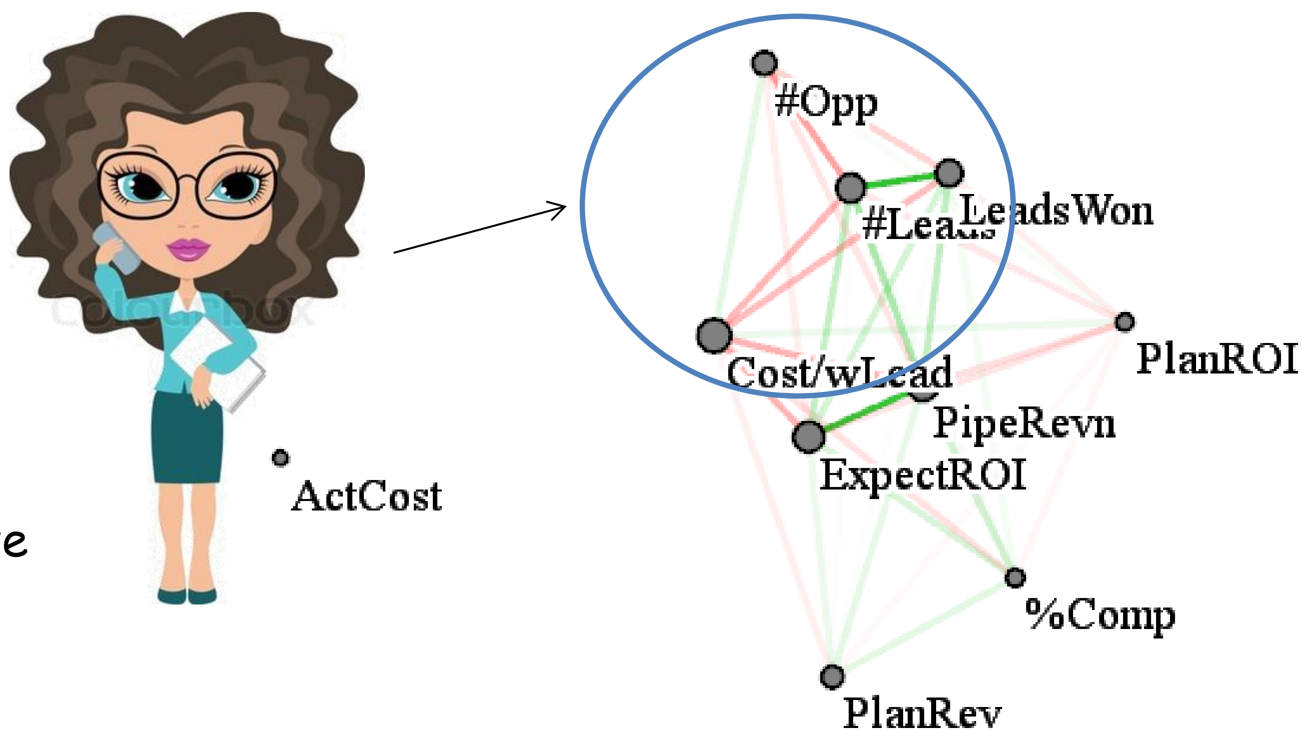
Evidence:

- data of three sales teams with a couple of hundred sales people in each team

Kate Explains It All

Meet Kate, a sales analyst in the meeting room:

> "OK…let's see, cost/won lead is nearby and it has a positive correlation with #opportunities but also a negative correlation with #won leads"

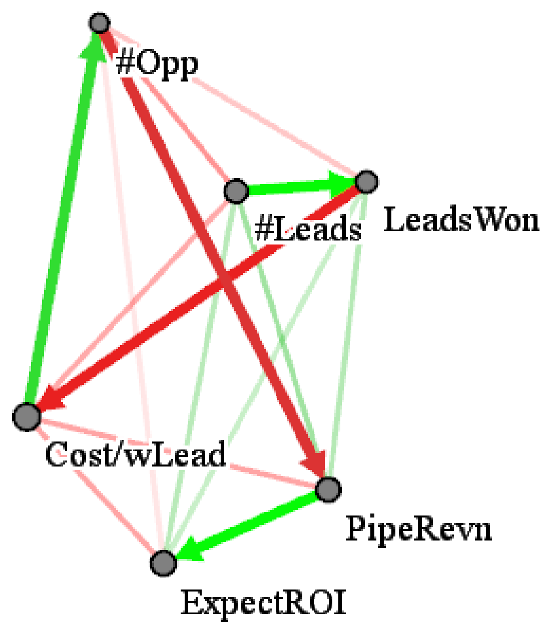


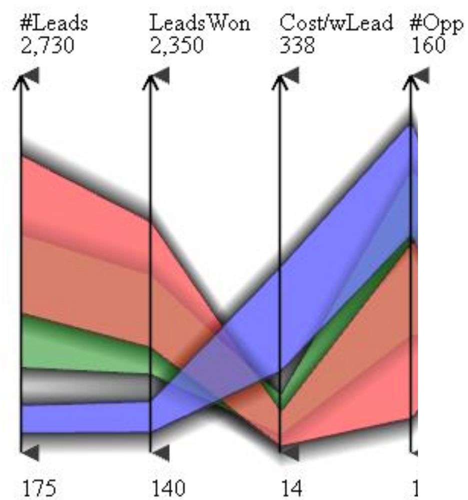
Kate

Kate Designs the Narration

"Let's go and make a revealing route!"

- she uses the mouse and designs the route shown
- she starts explaining the data like a story …





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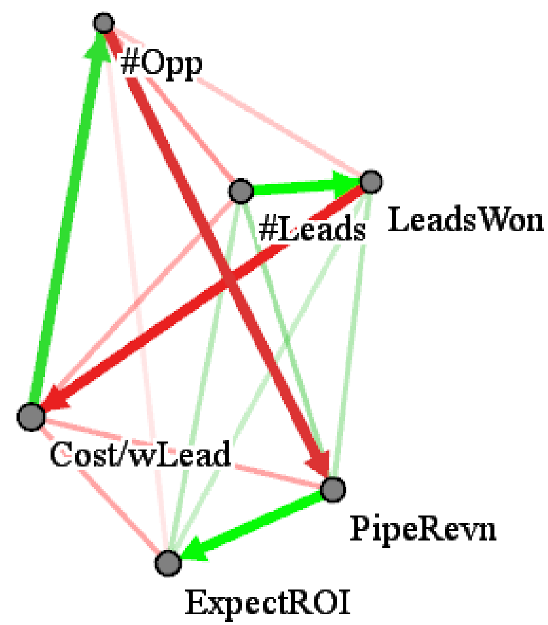
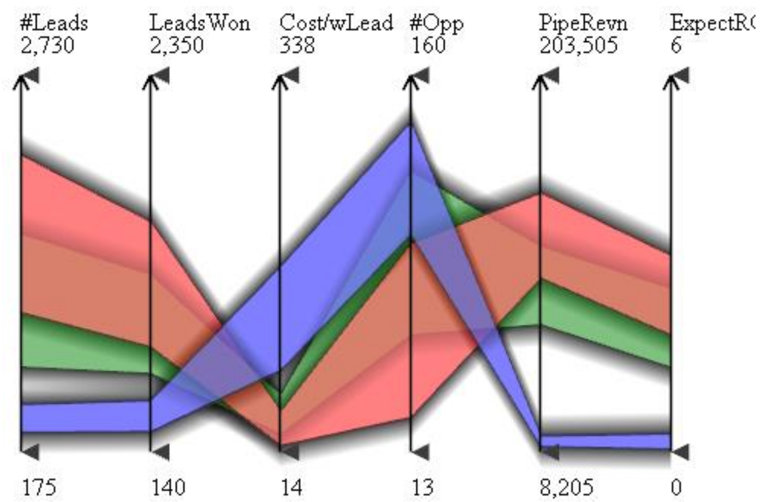
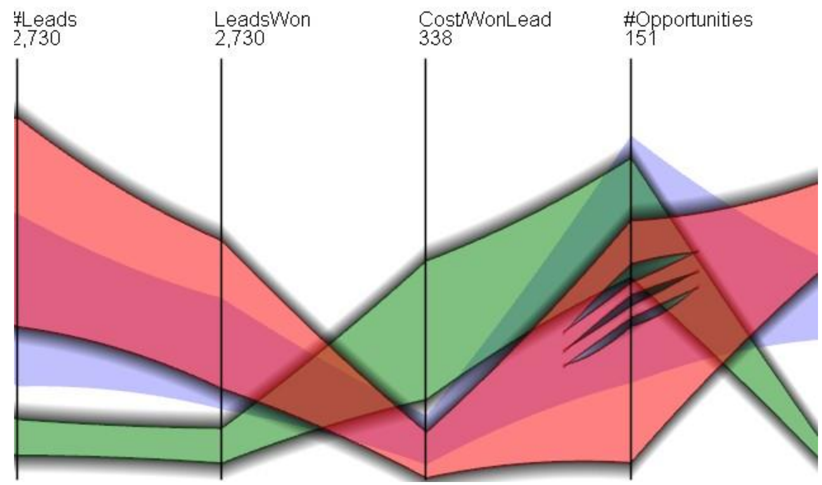




Further Insight



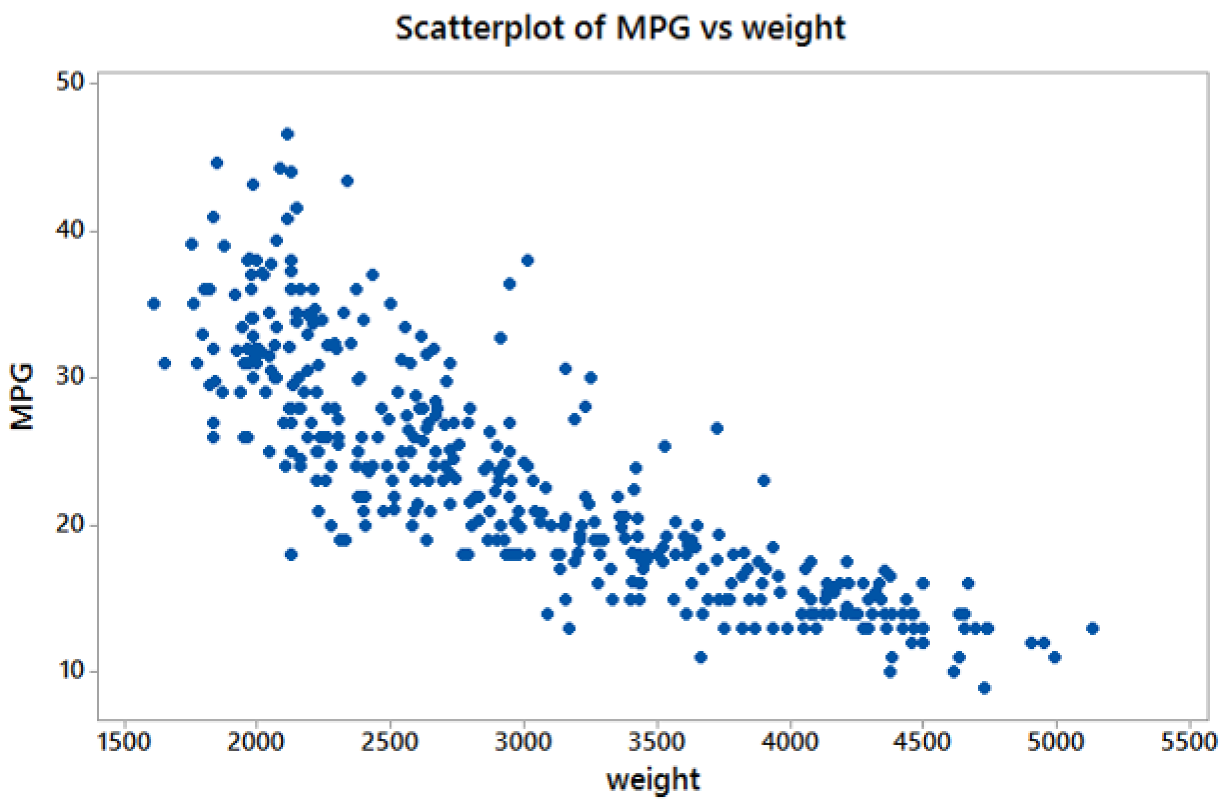
Kate notices something else:

- now looking at the red team
- there seems to be a spread in effectiveness among the team
- the team splits into three distinct groups

She recommends: "Maybe fire the least effective group or at least retrain them"

Scatterplots

Projection of the data items into a bivariate basis of axes



Projection Operations

How does 2D projection work in practice?

- N-dimensional point $x = \{x_1. x_2, x_3, \ldots x_N\}$
- a basis of two orthogonal axis vectors defined in N-D space

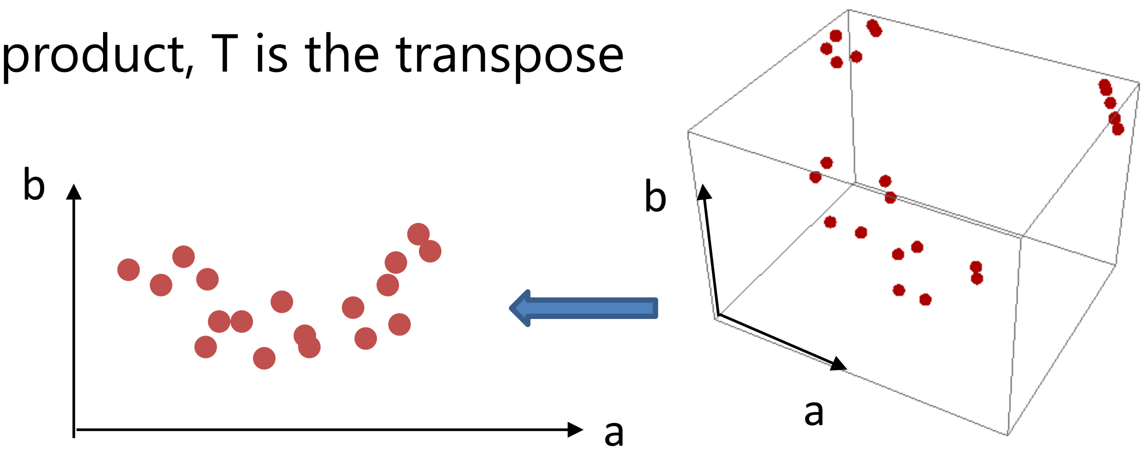
  $a = \{a_1. a_2, a_3, \ldots a_N\}$

  $b = \{b_1. b_2, b_3, \ldots b_N\}$
- a projection $\{x_a, x_b\}$ of x into the 2D basis spanned by {a, b} is:

  $x_a = a \cdot x^T$

  $x_b = b \cdot x^T$

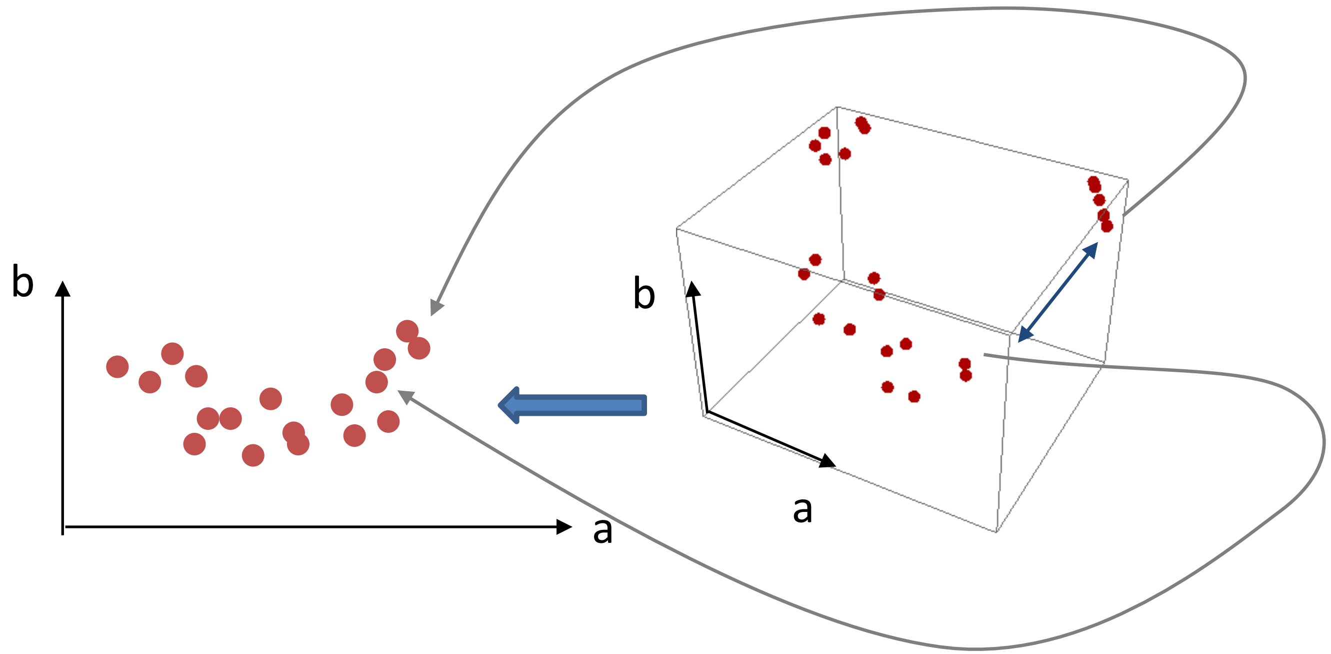
where · is the dot product, T is the transpose



Projection Ambiguity

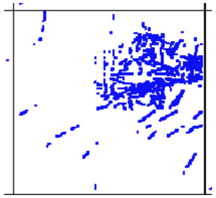
Projection causes inaccuracies

- close neighbors in the projections may not be close neighbors in the original higher-dimensional space
- this is called *projection ambiguity*



Scatterplot For Two Attributes

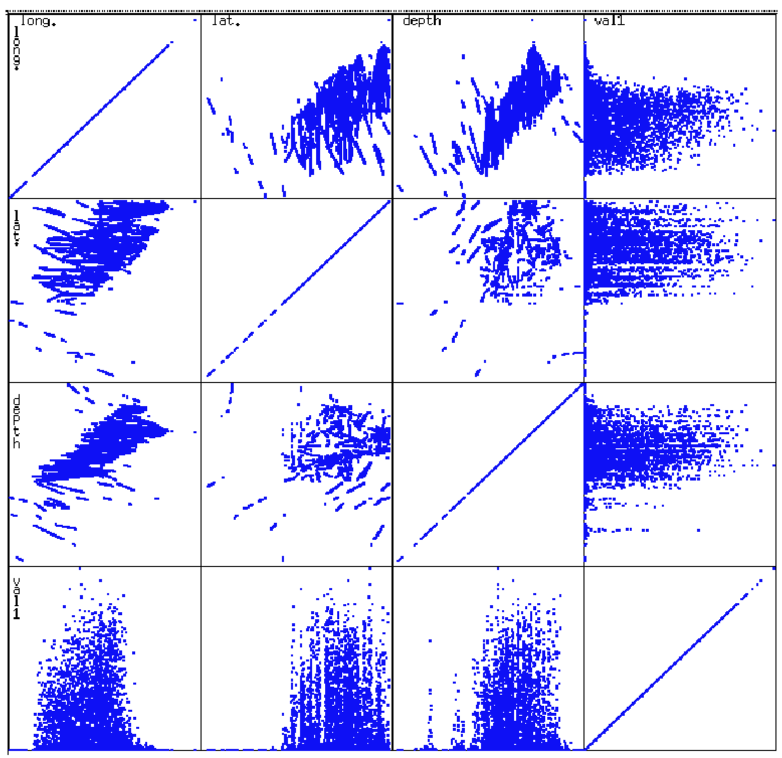
Appropriate for the display of bivariate relationships



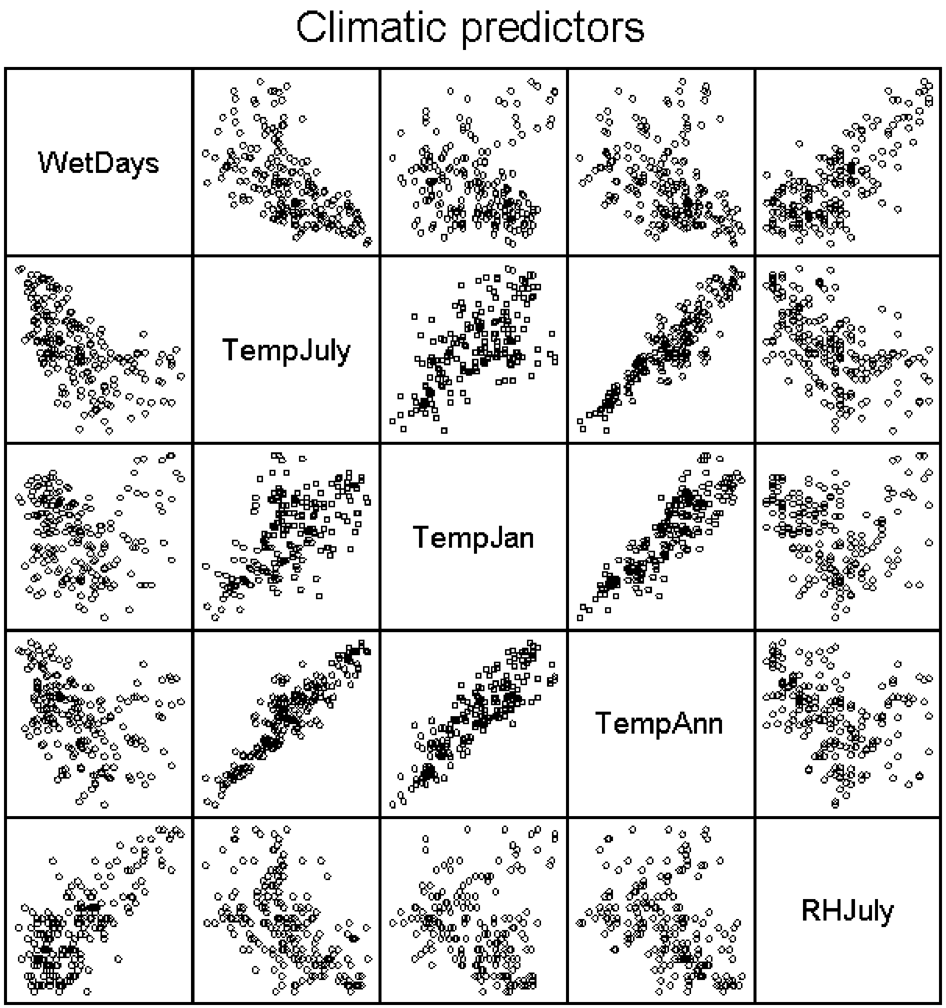
Scatterplot For many Attributes

What to do when there are more than two variables?

- arrange multivariate relationships into scatterplot matrices
- not overly intuitive to perceive multivariate relationships



Scatterplot Matrix (SPLOM)



Scatterplot Matrix

Scatterplot version of parallel coordinates

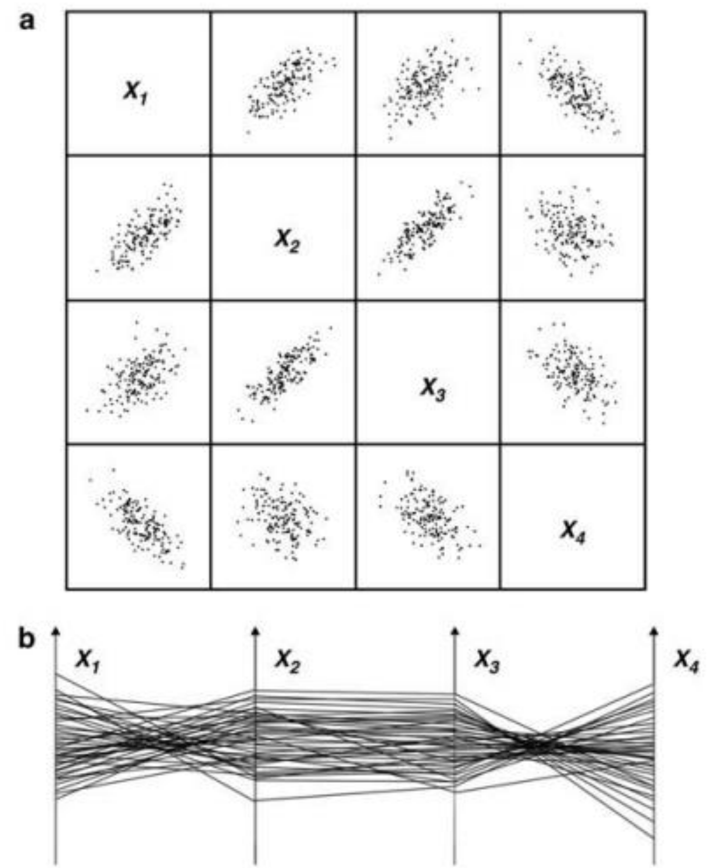
- distributes n(n-1) bivariate relationships over a set of tiles
- for n=4 get 16 tiles
- can use n(n-1)/2 tiles

For even moderately large n:

- there will be too many tiles

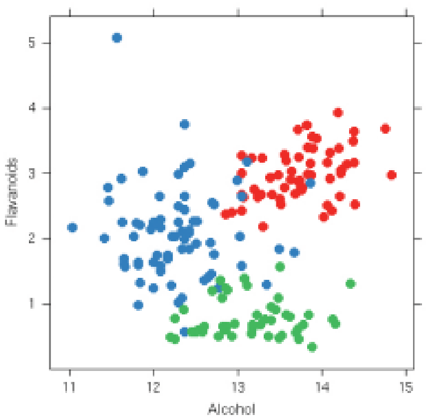
Which plots to select?

- plots that show correlations well
- plots that separate clusters well

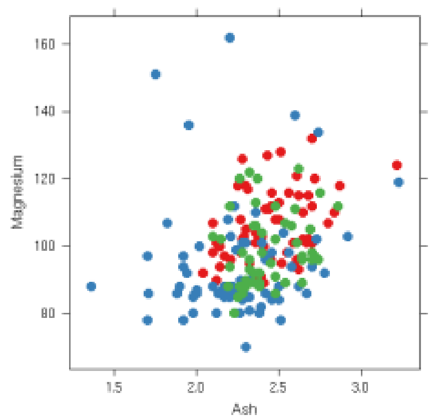


Automated Scatterplot Selection

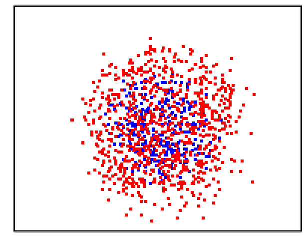
Several metrics, a good one is Distance Consistency (DSC)



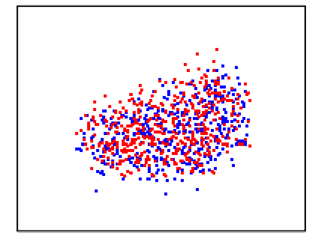
(a) **DSC=90**



(b) **DSC=49**

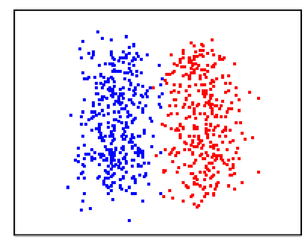


(d) 29

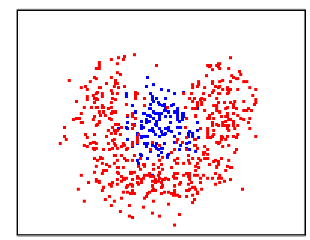


(e) 15

bad



(a) 99



(b) 74

OK

$$\mathbf{DSC} = \frac{\left| x' \in v(X) : \mathbf{CD}(x', centr'(c_{clabel(x)}) = true \right|}{k}$$

- measures how "pure" a cluster is
- pick the views with highest normalized DSC

M. Sips et al., Computer Graphics Forum, 28(3): 831–838, 2009

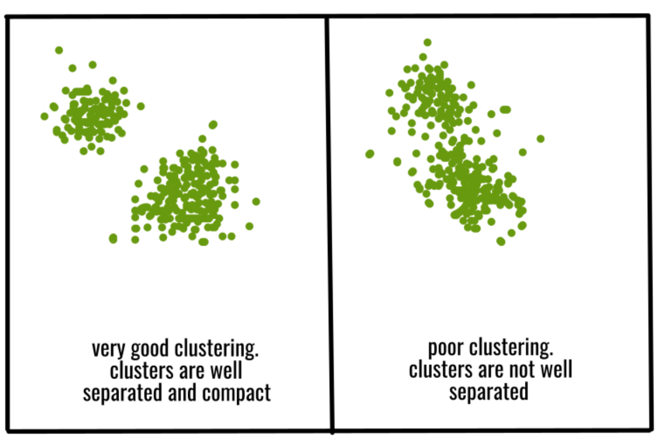
Dunn Index

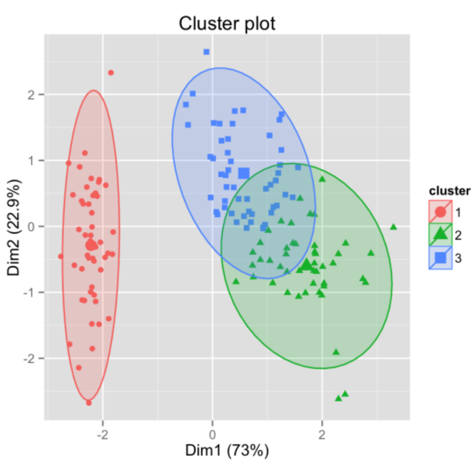
Favors clusters that are compact and are well isolated

$$DI_m = \frac{\min\limits_{1 \leqslant i < j \leqslant m} \delta(C_i, C_j)}{\max\limits_{1 \leqslant k \leqslant m} \Delta_k}.$$

$\Delta_i = \frac{\sum_{x \in C_i} d(x, \mu)}{|C_i|}, \mu = \frac{\sum_{x \in C_i} x}{|C_i|}$, calculates distance of all the points from the mean.

$\delta(C_i, C_j)$ be this intercluster distance metric, between clusters C_i and C_j.

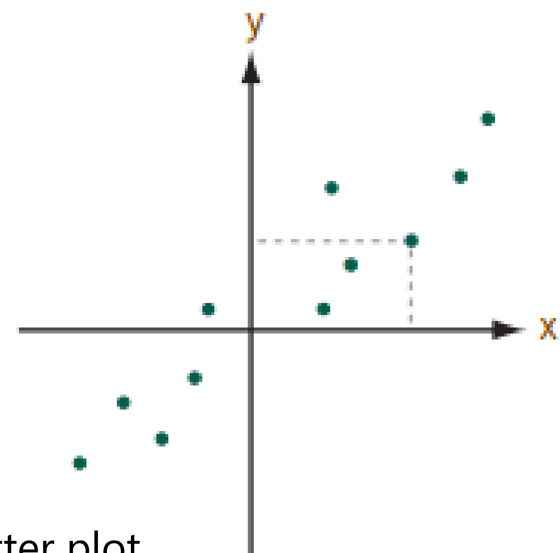




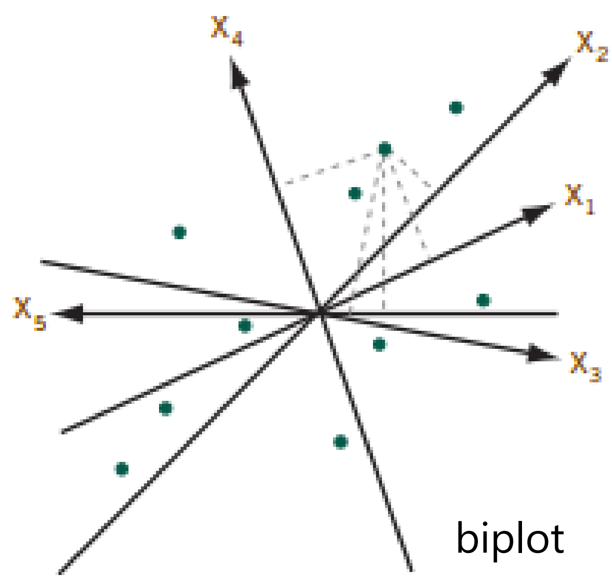
BIPLOTS

Plots data points and dimension axes into a single visualization

- uses first two PCA vectors as the basis to project into
- find plot coordinates [x] [y]
  - for data points: [PCA_1 · data vector] [PCA_2 · data vector]
  - for dimension axes: [PCA_1[dimension]] [PCA_2[dimension]]



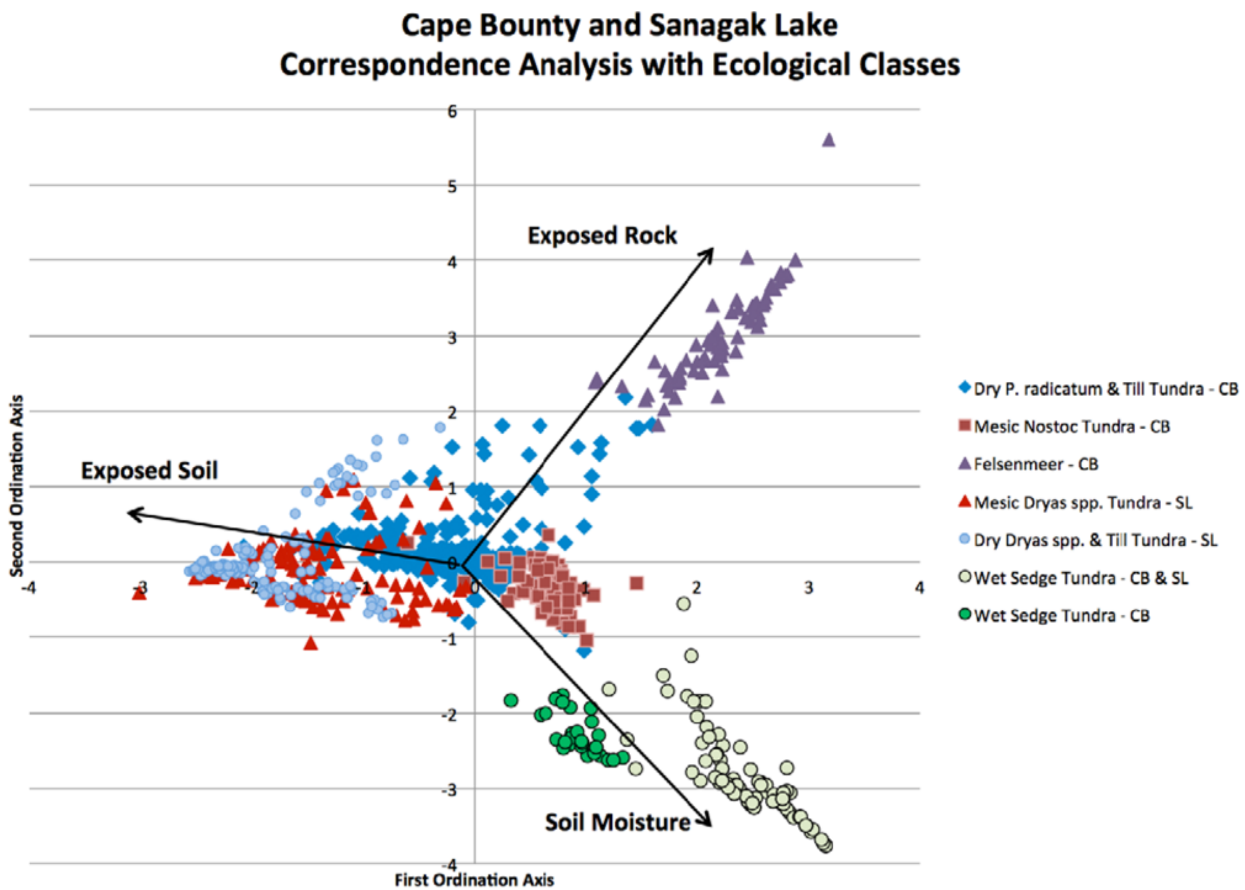
scatter plot



biplot

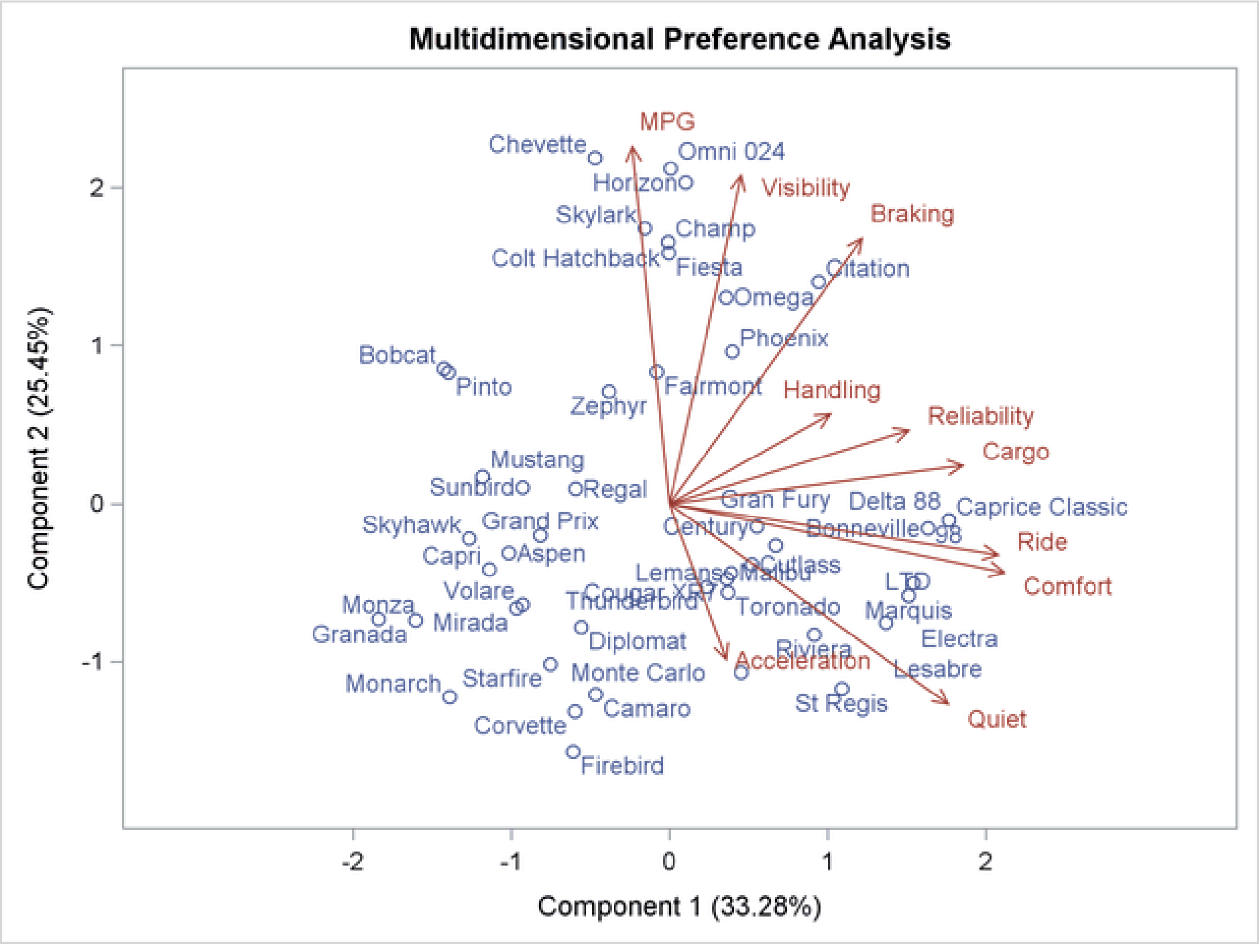
Biplots in Practice

See data distributions into the context of their attributes



Biplots in Practice

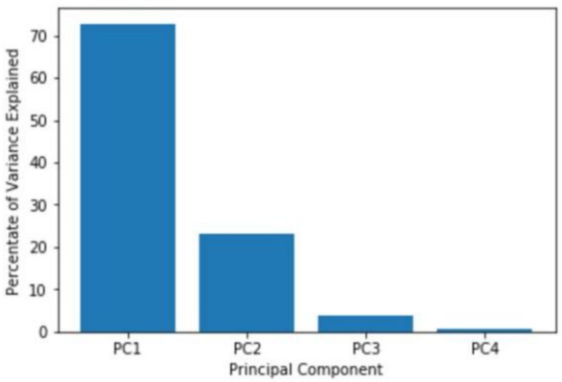
See data points into the context of their attributes



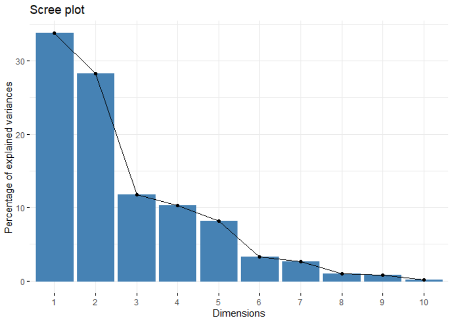
Biplots – A Word of caution

Do be aware that the projections may not be fully accurate

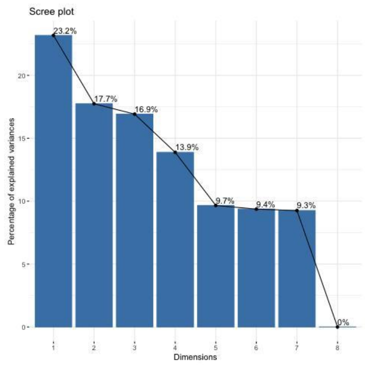
- you are projecting N-D into 2D by a linear transformation
- if there are more than 2 significant PCA vectors then some variability will be lost and won't be visualized
- remote data points might project into nearby plot locations suggesting false relationships → projection ambiguity
- always check out the PCA scree plot to gauge accuracy



OK



OK

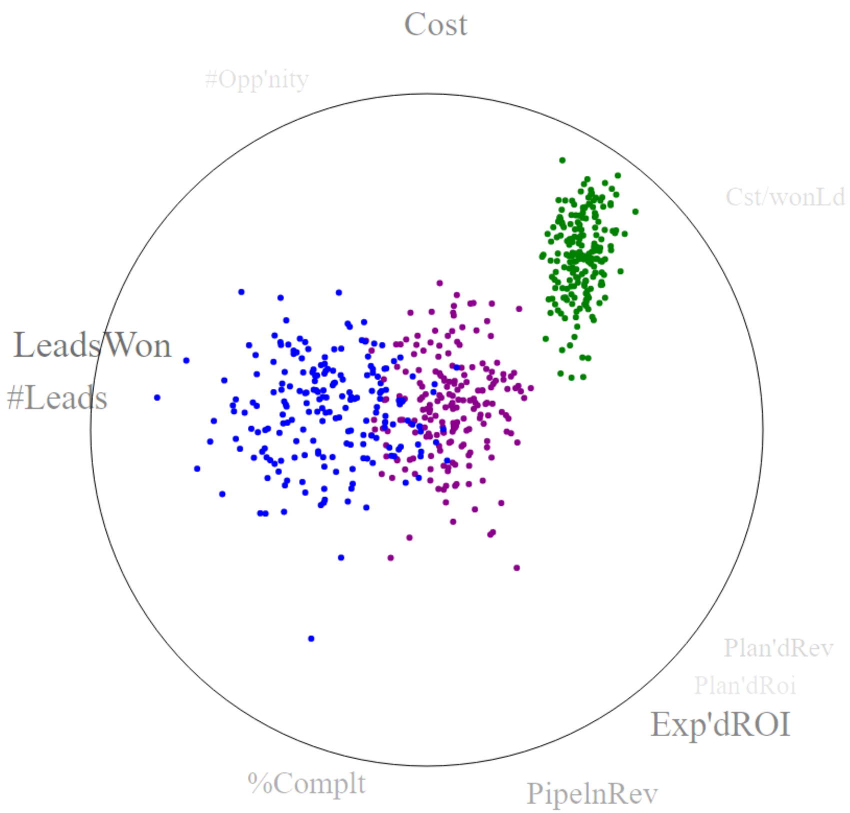


be careful

Interactive Biplots

Also called multivariate scatterplot

- biplot-axes length vis replaced by graphical design
- less cluttered view
- but there's more to this .....



Meet the *Subspace Voyager*

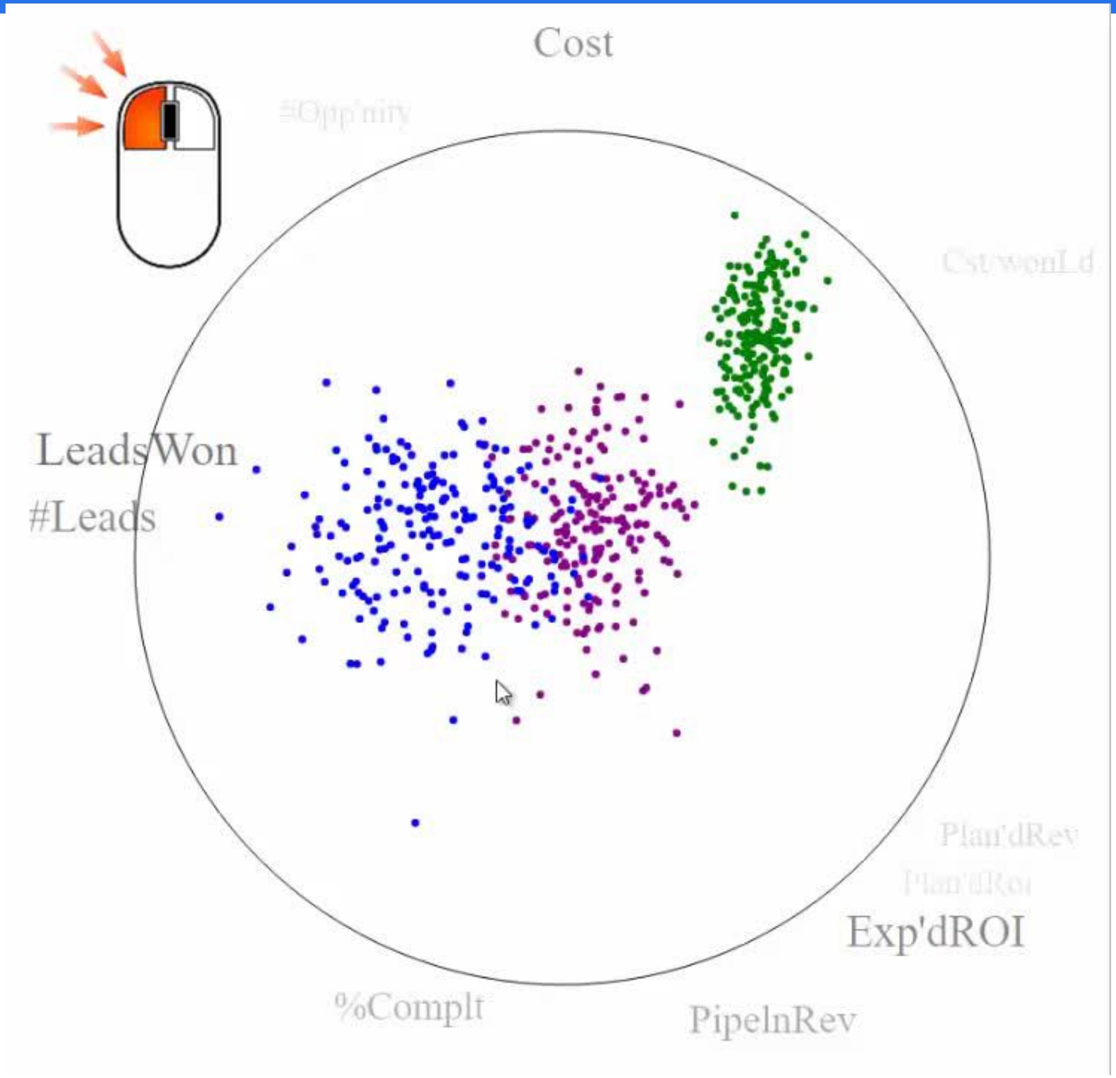
Decomposes high-D data spaces into lower-D subspaces by

- clustering
- classification
- reducing clusters to intrinsic dimensionality via local PCA

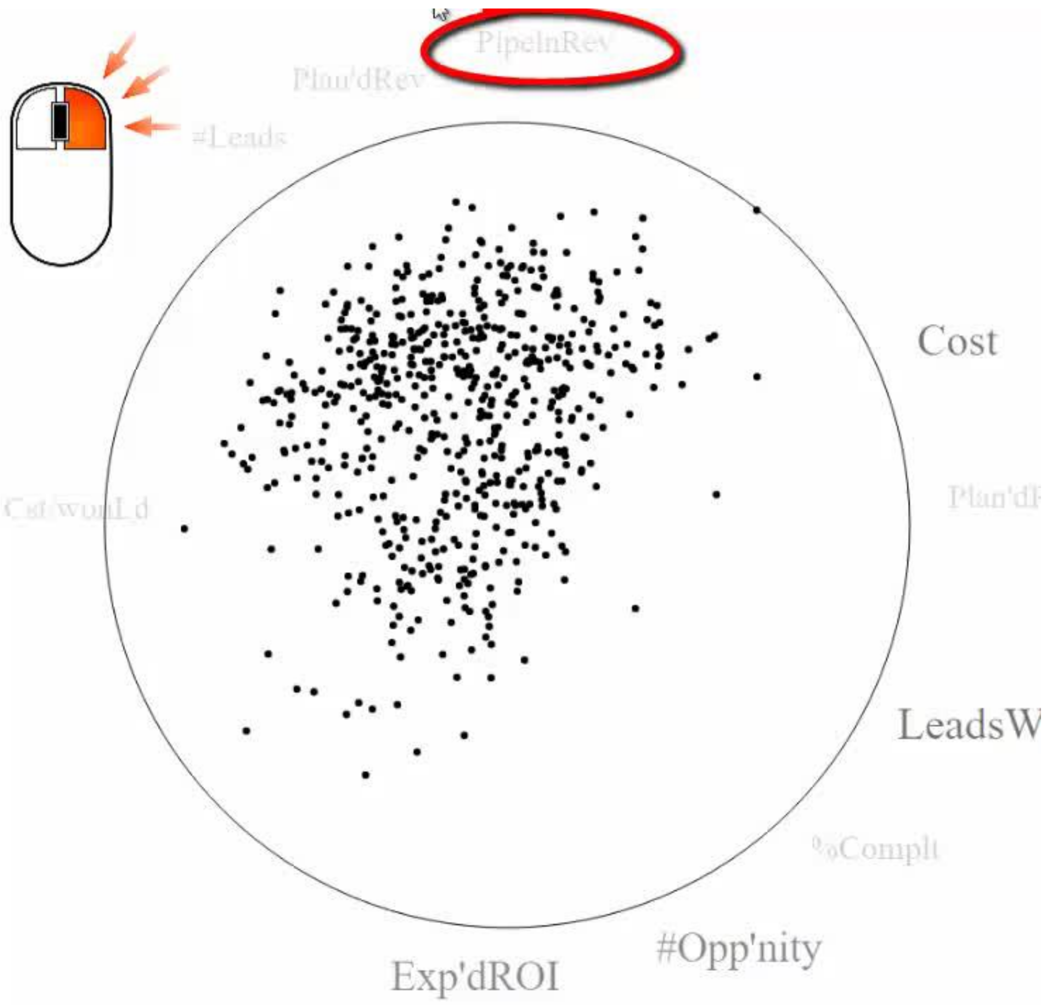
Allows users to interactively explore these lower-D subspaces

- explore them as a chain of 3D subspaces
- transition seamlessly to adjacent 3D subspaces on demand
- save observations as you go (and return to them just as well)

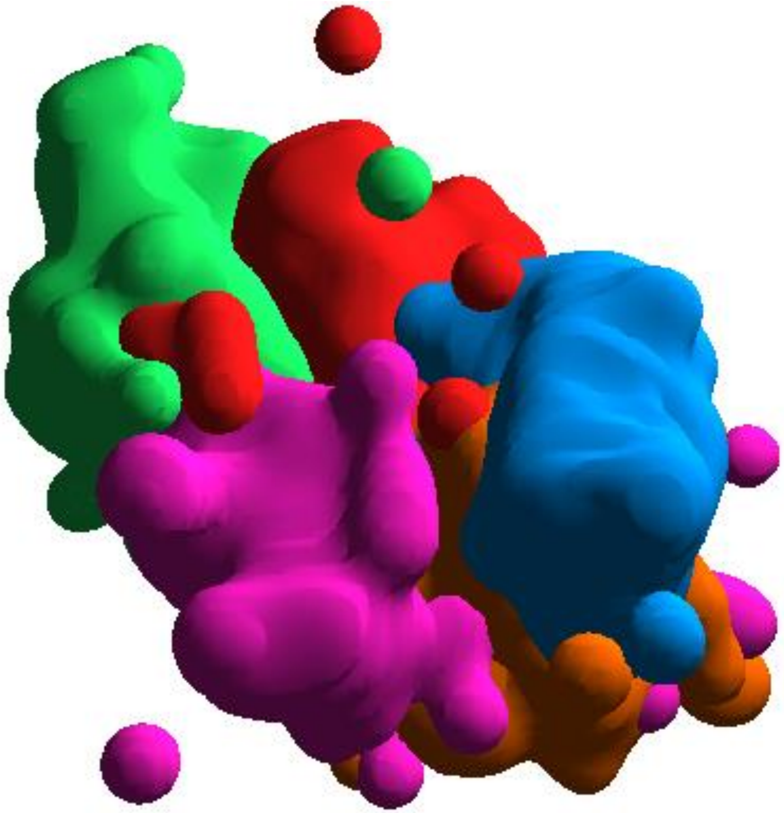
Trackball-Based Cluster Exploration



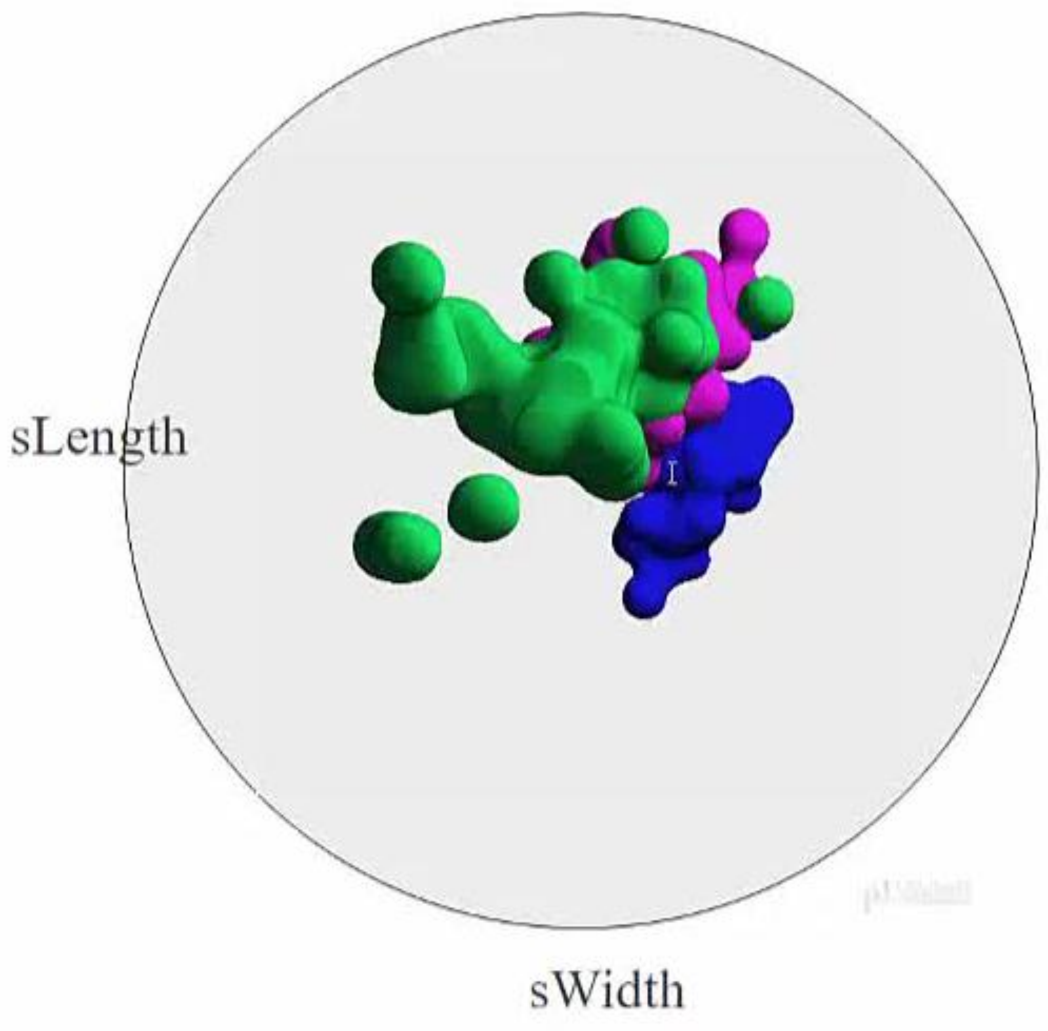
Chase Interesting Clusters – Transition to Adjacent 3D Subspaces



Clarify Spatial Relationships



Clarify Spatial Relationships



Star Coordinates

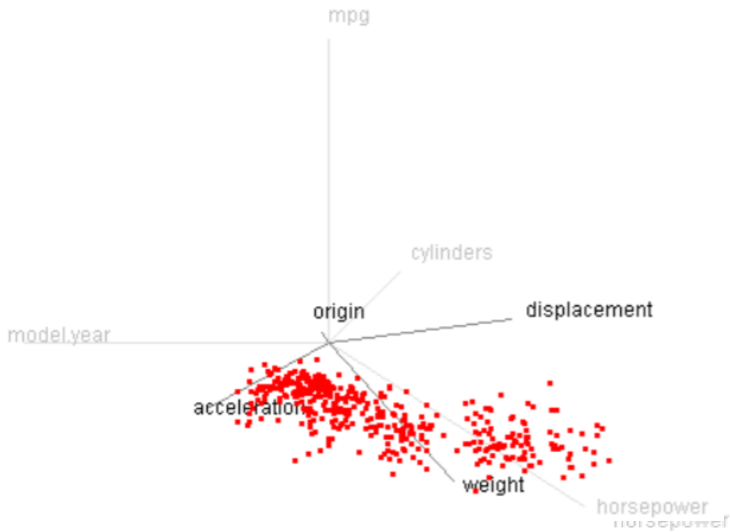
Coordinate system based on axes positioned in a star

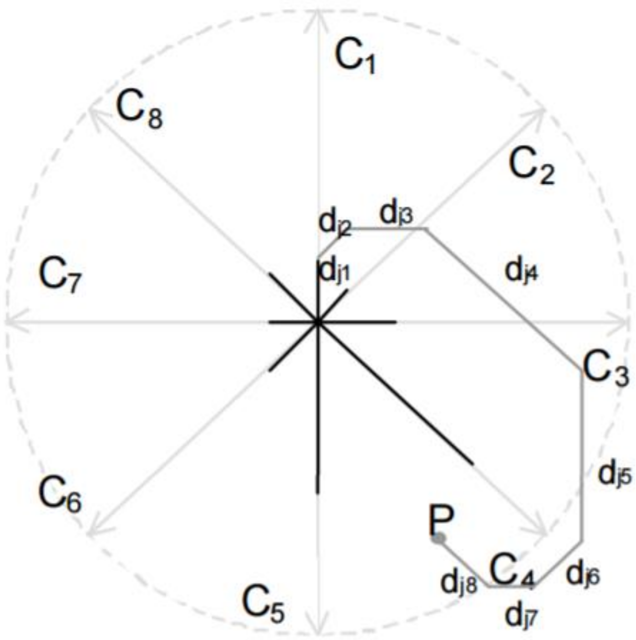
- a point P is vector sum of all axis coordinates

$$P = O + \sum_{i=1}^{m} d_i \vec{c}_i$$

Interactions

- axis rescaling, rotation
- reveal correlations
- resolve plotting ambiguities



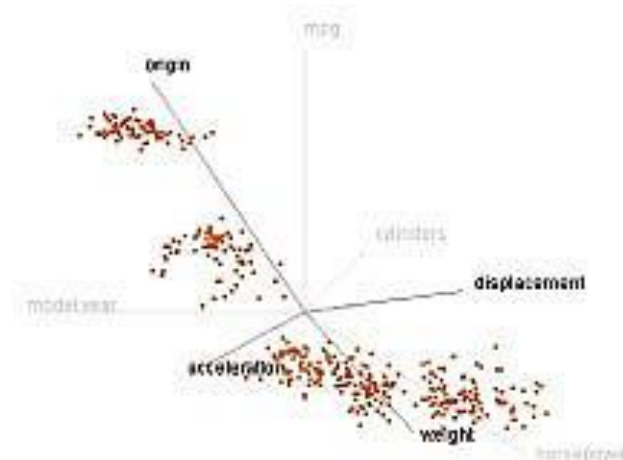


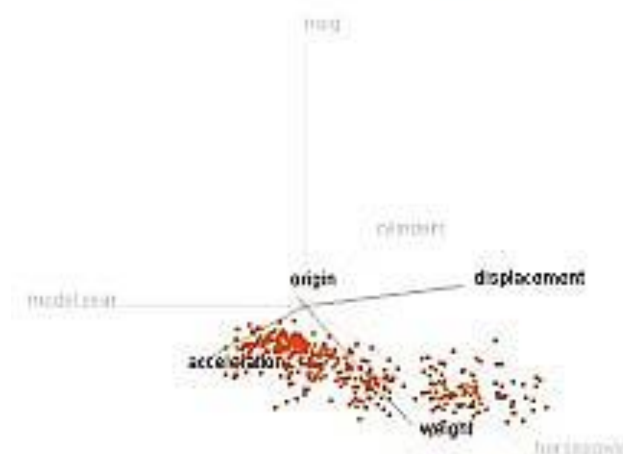
[E. Kandogan SIGKDD 2001]

Star Coordinates

Operations defined on Star Coords

- scaling changes contribution to resulting visualization
- axis rotation can visualize correlations
- also used to reduce projection ambiguities





RadViz

Similar to Star Coordinates

- uses a spring model
- difference is normalization by sum of values

$$P = \frac{\sum_{i=1}^{m} d_i \vec{c}_i}{\sum_{i=1}^{m} d_i}$$

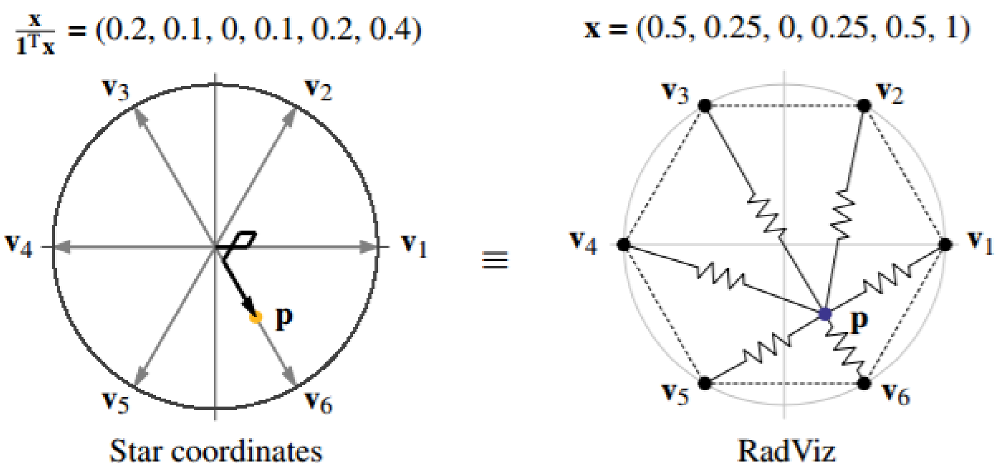
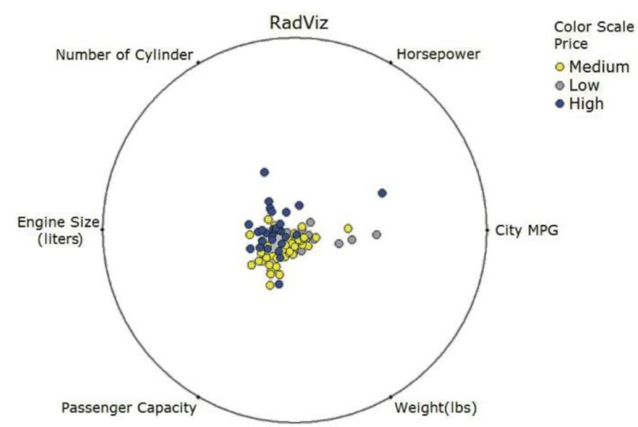


Figure by: Rubio-Sanchez et al. TVCG 2015



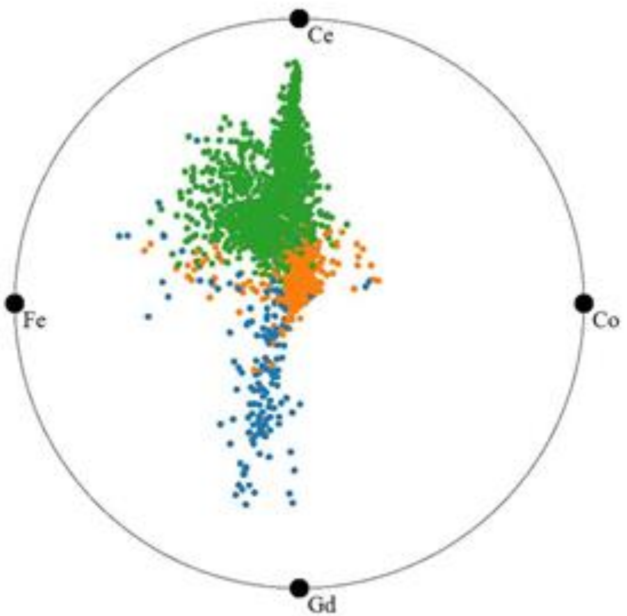
[P. Hoffman et al. VIS 1997]

Optimizing the RadViz Layout

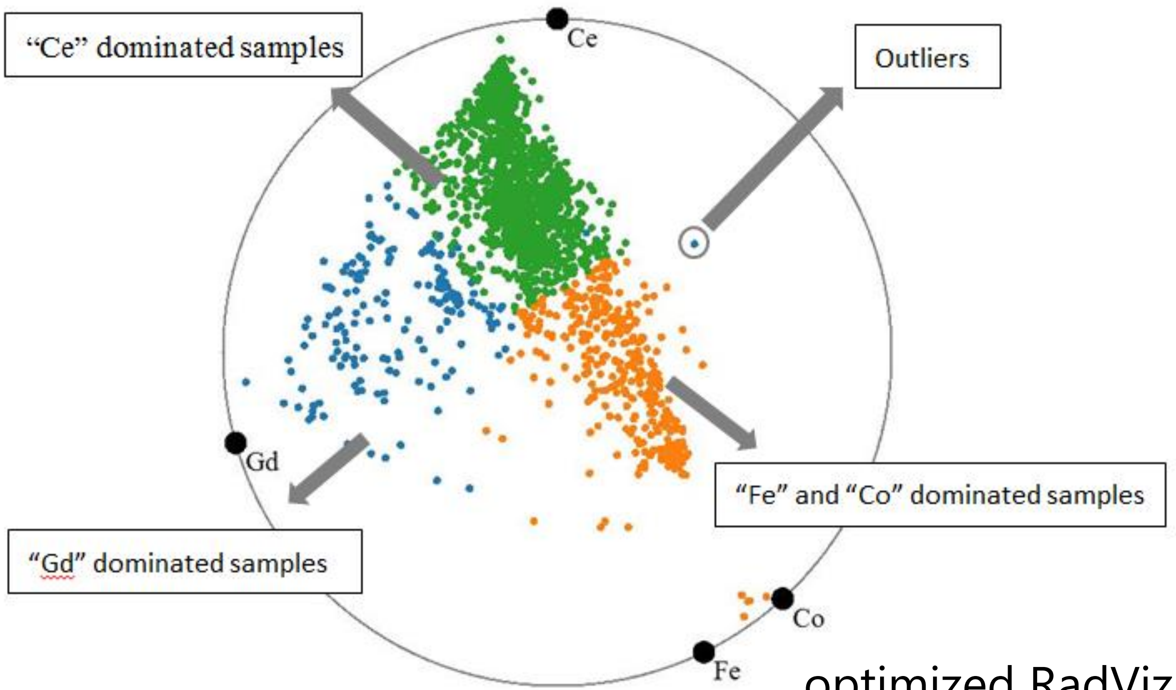
[Cheng and Mueller, Pacific Vis 2015]

Optimize

- correlation-based attribute placement on circle using TSP
- samples placed iteratively into circle using similarity constraints



standard RadViz

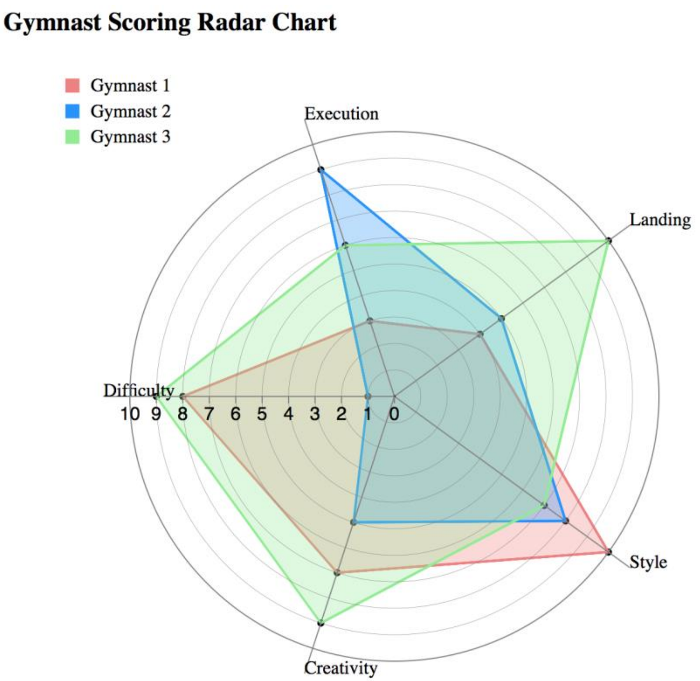


optimized RadViz

Radar Chart

Equivalent to a parallel coordinates plot, with the axes arranged radially

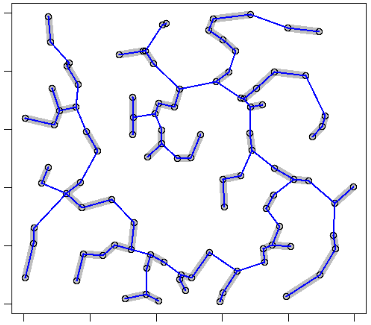
- each star represents a single observation
- can show outliers and commonalities nicely



Disadvantages

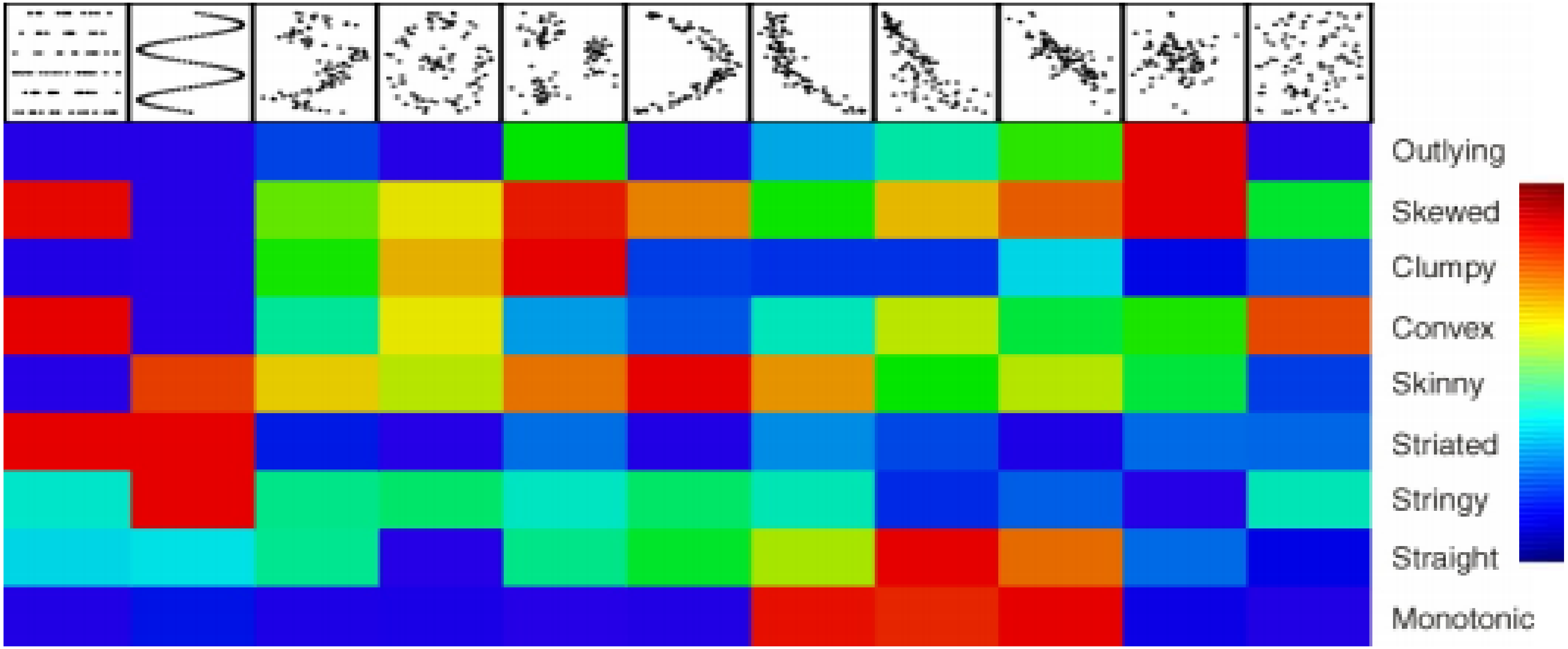
- hard to make trade-off decisions
- distorts data to some extent when lines are filled in

Scagnostics



Describe scatterplot features by graph theoretic measures

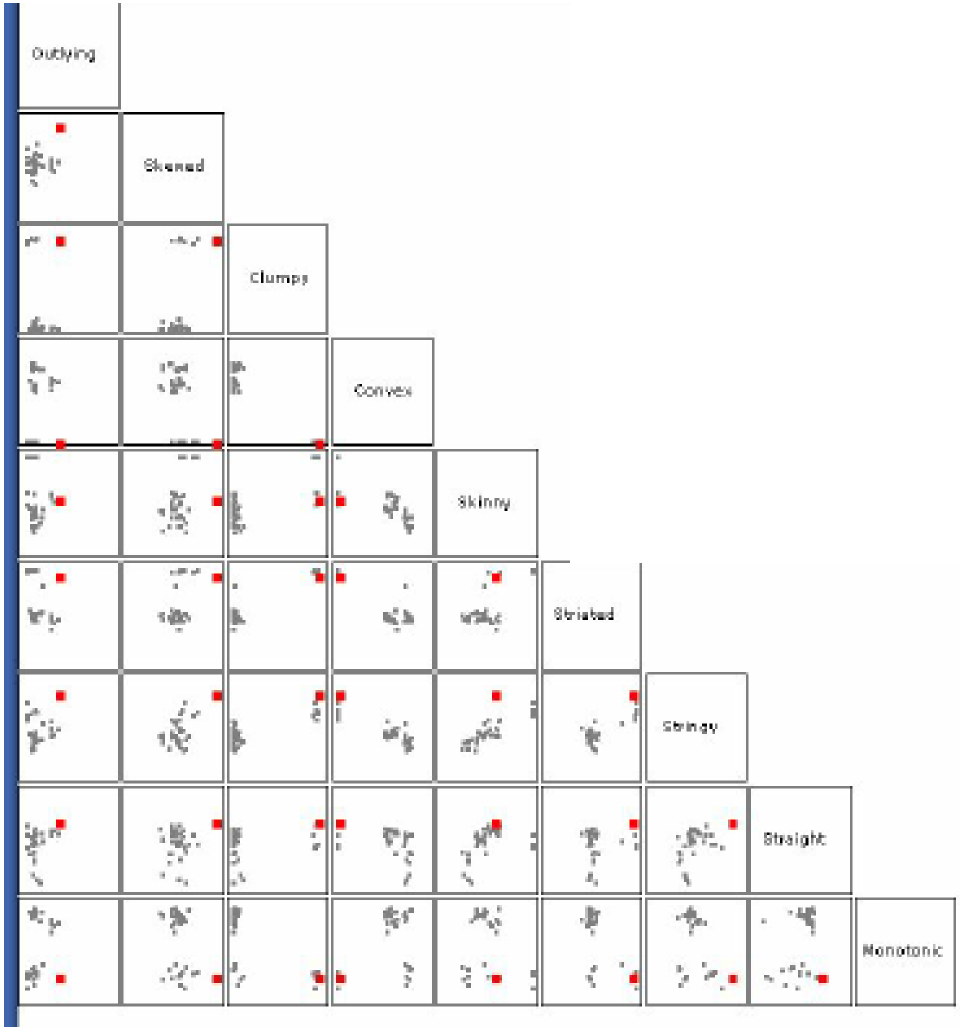
- mostly built on minimum spanning tree
- can be used to summarize large sets of scatterplots



Scatterplot of Scatterplots

Use scagnostics to quickly survey 1,000s of scatterplots

- compute scagnostics measures
- create scatterplot matrix of these measures
- each scatterplot is a point



Commonalities

All of these scatterplot displays share the following characteristics

- allow users to see the data points in the context of the variables
- but can suffer from projection ambiguity
- some offer interaction to resolve some of these shortcomings
- but interaction can be tedious

Are there visualization paradigms that can overcome these problems?

- yes, algorithms that optimize the layout to preserve distances or similarities in high-dimensional space
- as opposed to the linear schemes we discussed so far, these are non-linear embedding algorithms

Multidimensional Scaling (MDS)

MDS is for irregular structures

- scattered points in high-dimensions (N-D)
- adjacency matrices

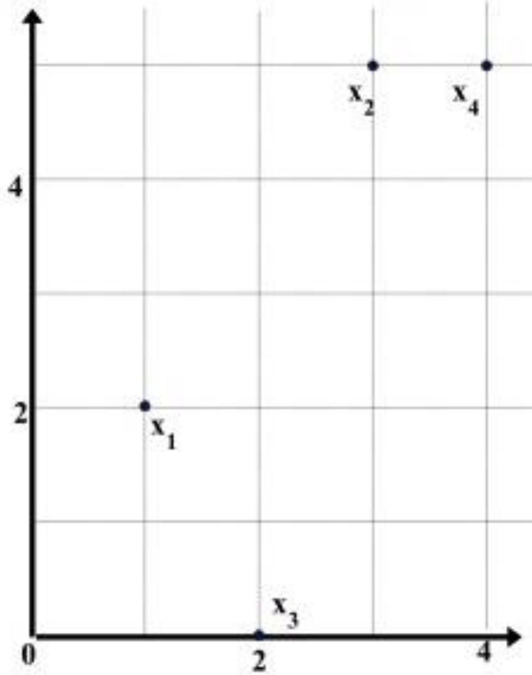
Maps the distances between observations from N-D into low-D (say 2D)

- attempts to ensure that differences between pairs of points in this reduced space match as closely as possible

The input to MDS is a distance (similarity) matrix

- actually, you use the *dissimilarity* matrix because you want similar points mapped closely
- dissimilar point pairs will have greater values and map father apart

The Dissimilarity Matrix



Data Matrix

point	attribute1	attribute2
x1	1	2
x2	3	5
x3	2	0
x4	4	5

Dissimilarity Matrix

(with Euclidean Distance)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Distance Matrix

MDS turns a distance matrix into a network or point cloud

- correlation, cosine, Euclidian, and so on

Suppose you know a matrix of distances among cities

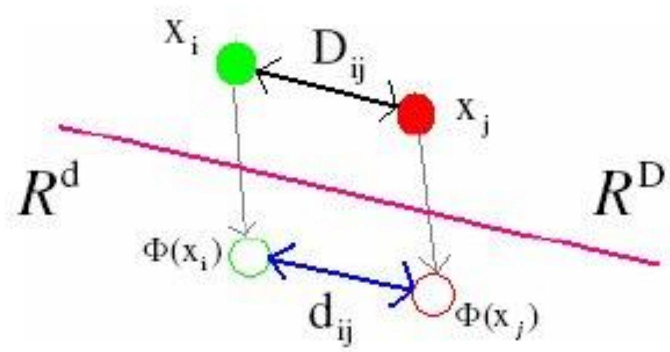
	Chicago	Raleigh	Boston	Seattle	S.F.	Austin	Orlando
Chicago	0						
Raleigh	641	0					
Boston	851	608	0				
Seattle	1733	2363	2488	0			
S.F.	1855	2406	2696	684	0		
Austin	972	1167	1691	1764	1495	0	
Orlando	994	520	1105	2565	2458	1015	0

Result of MDS

Compare with real Map

MDS Algorithm

- Task:
  - Find that configuration of image points whose pairwise distances are most similar to the original inter-point distances !!!
- Formally:
  - Define: $D_{ij} = \| x_i - x_j \|_D$ $\quad d_{ij} = \| y_i - y_j \|_d$
  - Claim: $D_{ij} \equiv d_{ij} \quad \forall i, j \in [1, n]$
- In general: an exact solution is not possible !!!
- Inter Point distances → invariance features



MDS Algorithm

<u>Strategy (of metric MDS):</u>

- iterative procedure to find a good configuration of image points
  - 1) Initialization
    → Begin with some (arbitrary) initial configuration
  - 2) Alter the image points and try to find a configuration of points that minimizes the following sum-of-squares error function:

MDS Algorithm

Strategy (of metric MDS):

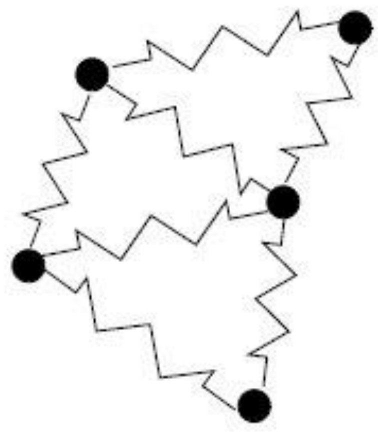
- iterative procedure to find a good configuration of image points
  - 1) Initialization
    → Begin with some (arbitrary) initial configuration
  - 2) Alter the image points and try to find a configuration of points that minimizes the following sum-of-squares error function:

$$E = \sum_{i<j}^{N} \left(D_{ij} - d_{ij}\right)^2$$

Force-Directed Algorithm

Spring-like system

- insert springs within each node
- the length of the spring encodes the desired node distance
- start at an initial configuration
- iteratively move nodes until an energy minimum is reached



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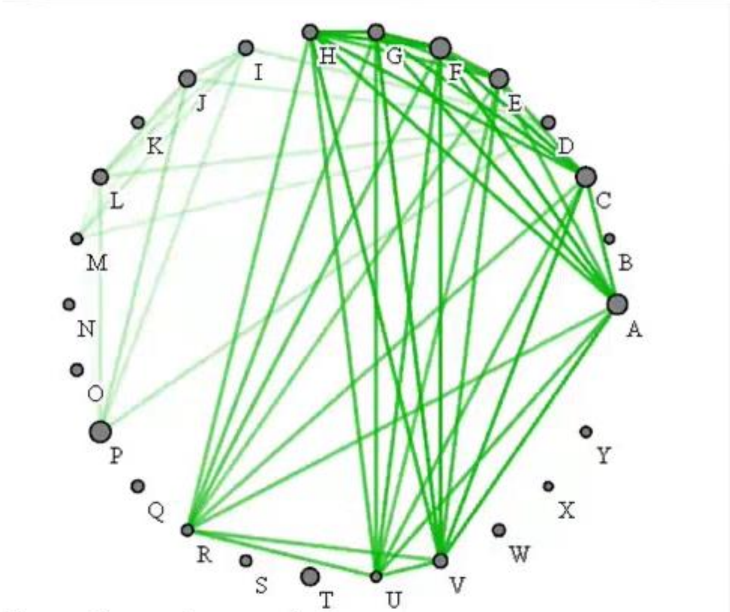


Vertex layout by correlations.

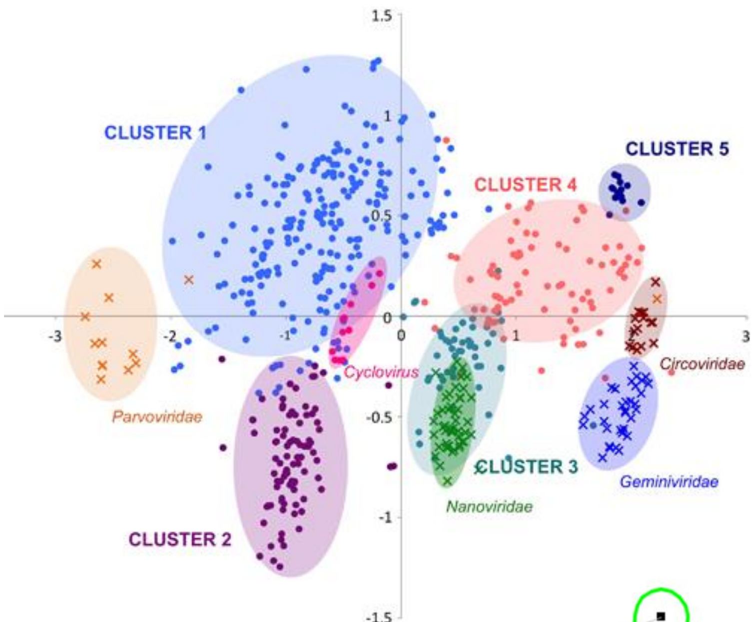
Uses of MDS

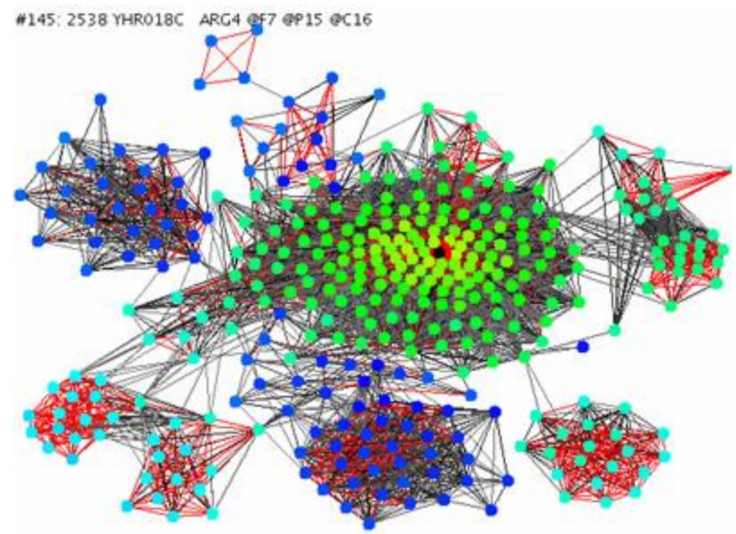
Distance (similarity) metric

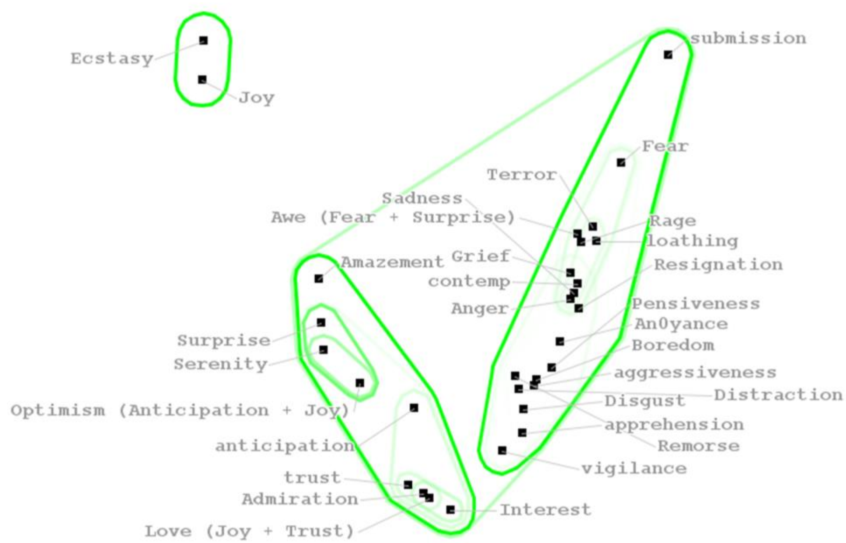
- Euclidian distance (best for data)
- Cosine distance (best for data)
- |1-correlation| distance (best for attributes)
- use 1-correlation to move correlated attribute points closer
- use | | if you do not care about positive or negative correlations

	MRK	MSFT	PFE	PG	T	TRV	UTX	VZ	WMT	XOM
MRK	1.	0.39	0.72	−0.43	0.57	0.031	−0.26	0.61	−0.11	−0.25
MSFT	0.39	1.	0.14	0.11	0.56	0.25	0.25	0.67	−0.074	0.24
PFE	0.72	0.14	1.	−0.77	0.08	−0.37	−0.65	0.19	−0.077	−0.72
PG	−0.43	0.11	−0.77	1.	0.25	0.68	0.92	0.086	0.072	0.9
T	0.57	0.56	0.08	0.25	1.	0.65	0.46	0.87	−0.059	0.54
TRV	0.031	0.25	−0.37	0.68	0.65	1.	0.83	0.43	−0.0067	0.81
UTX	−0.26	0.25	−0.65	0.92	0.46	0.83	1.	0.27	−0.033	0.93
VZ	0.61	0.67	0.19	0.086	0.87	0.43	0.27	1.	0.026	0.36
WMT	−0.11	−0.074	−0.077	0.072	−0.059	−0.0067	−0.033	0.026	1.	0.032
XOM	−0.25	0.24	−0.72	0.9	0.54	0.81	0.93	0.36	0.032	1.

MDS Examples

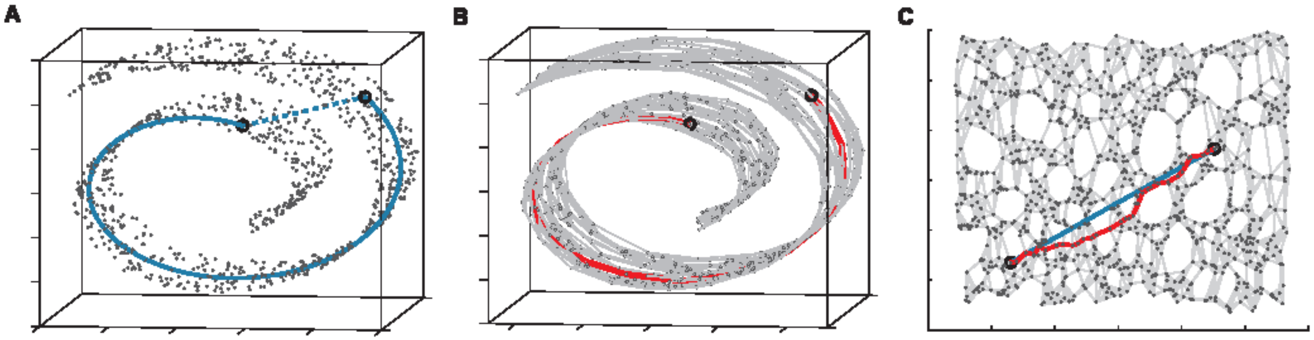






Manifold Learning: Isomap

by: [J. Tenenbaum, V. de Silva, J. Langford, Science, 2000]



Tries to unwrap a high-dimensional surface (A) → manifold

- noisy points could be averaged first and projected onto the manifold

Algorithm

- construct neighborhood graph G → (B)
- for each pair of points in G compute the shortest path distances → geodesic distances
- fill similarity matrix with these geodesic distances
- embed (layout) in low-D (2D) with MDS → (C)
- visualize it like an MDS layout

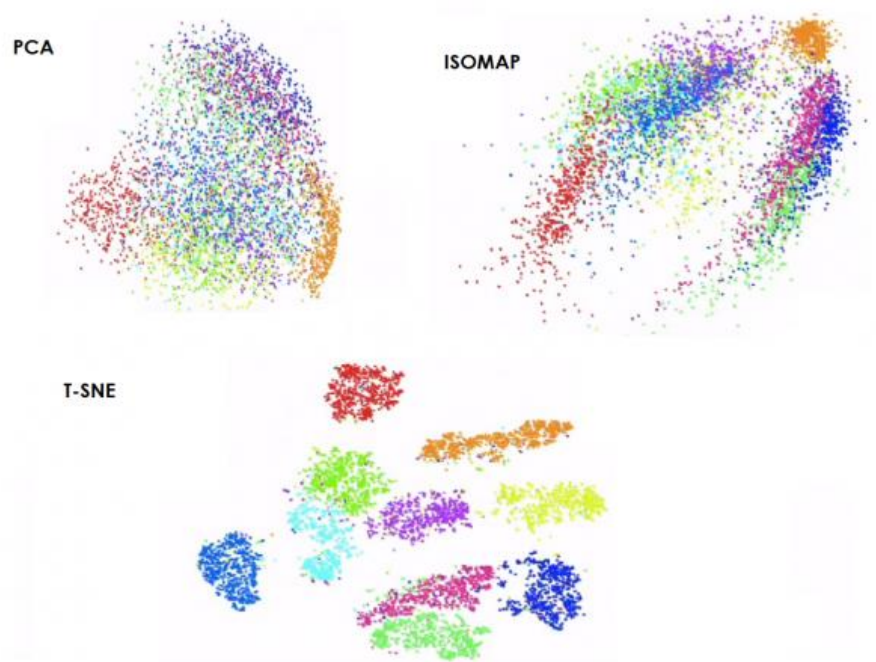
T-SNE

t-Distributed Stochastic Neighbor Embedding

- innovated by [l. van der Maaten and G. Hinton, 2008]

Works as a two-stage approach

1. Construct a probability distribution over pairs of high-D points based on similarity
2. Define a similar probability distribution over the points in the low-D map



Self-Organizing Maps (SOM)

Introduced by [T. Kohonen et al. 1996]

- unsupervised learning and clustering algorithm
- has advantages compared to hierarchical clustering
- often realized as an artificial neural network

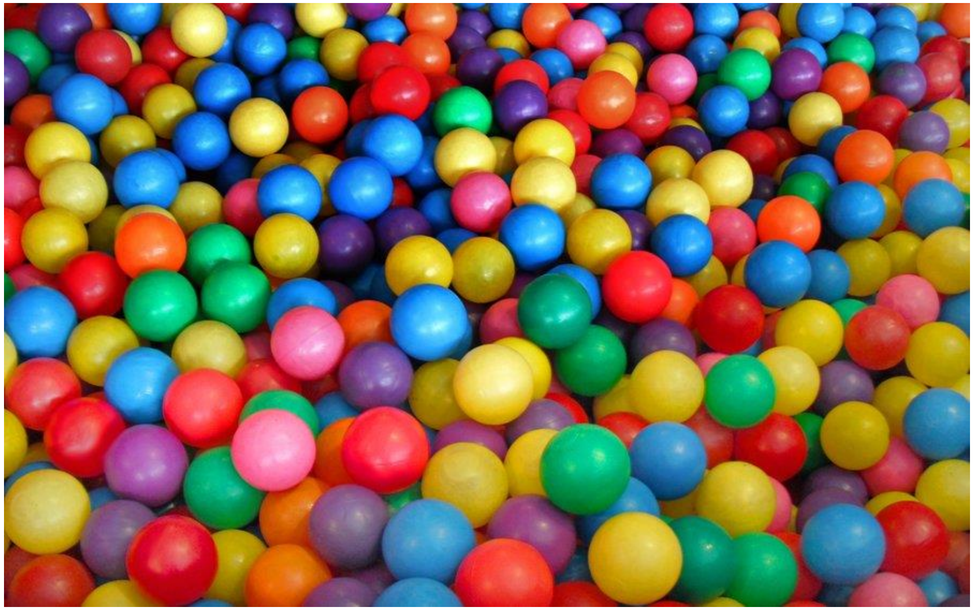
SOMs group the data

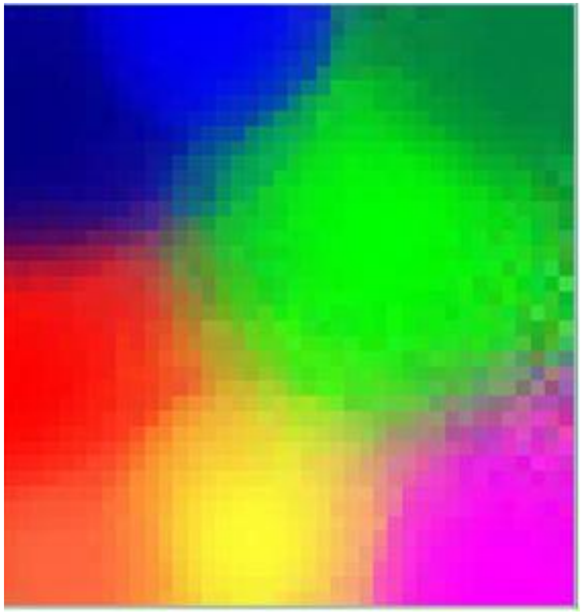
- perform a nonlinear projection from N-dimensional input space onto two-dimensional visualization space
- provide a useful topological arrangement of information objects in order to display clusters of similar objects in information space

SOM Example

Map a dataset of 3D color vectors into a 2D plane

- assume you have an image with 5 colors
- want to see how many there are of each
- compute an SOM of the color vectors

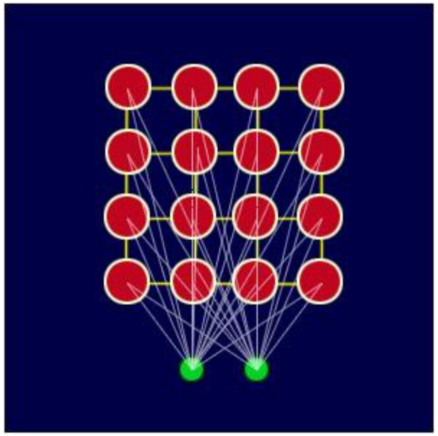




SOM Algorithm

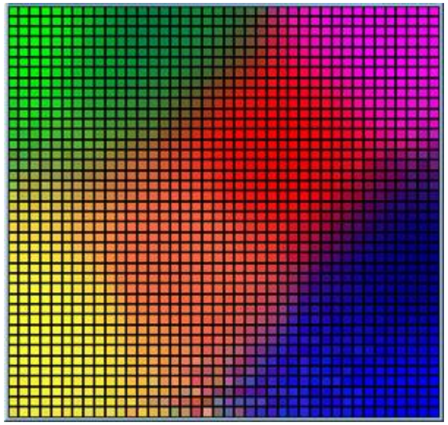
Create array and connect all elements to the N input dimensions

- shown here: 2D vector with 4×4 elements
- initialize weights



For each input vector chosen at random

- find node with weights most like the input vector
- call that node the Best Matching Unit (BMU)
- find nodes within neighborhood radius *r* of BMU
  - initially *r* is chosen as the radius of the lattice
  - diminishes at each time step
- adjust the weights of the neighboring nodes to make them more like the input vector
  - the closer a node is to the BMU, the more its weights get altered

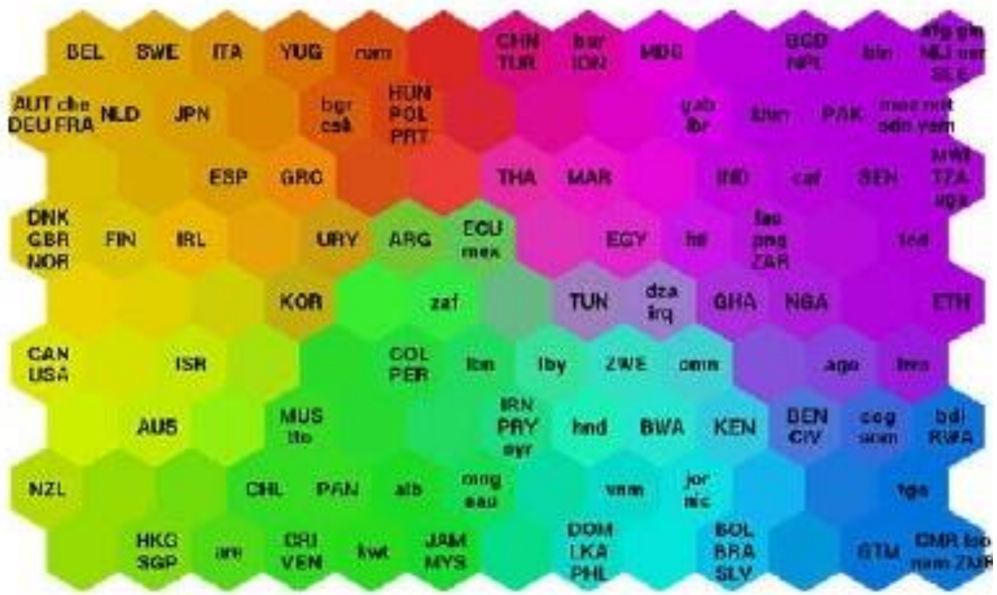


SOM Example: Poverty Map

SOM – Result Example

World Poverty Map

A SOM has been used to classify statistical data describing various quality-of-life factors such as state of health, nutrition, educational services etc. . **Countries with similar quality-of-life factors end up clustered together**. The countries with better quality-of-life are situated toward the upper left and the most poverty stricken countries are toward the lower right.

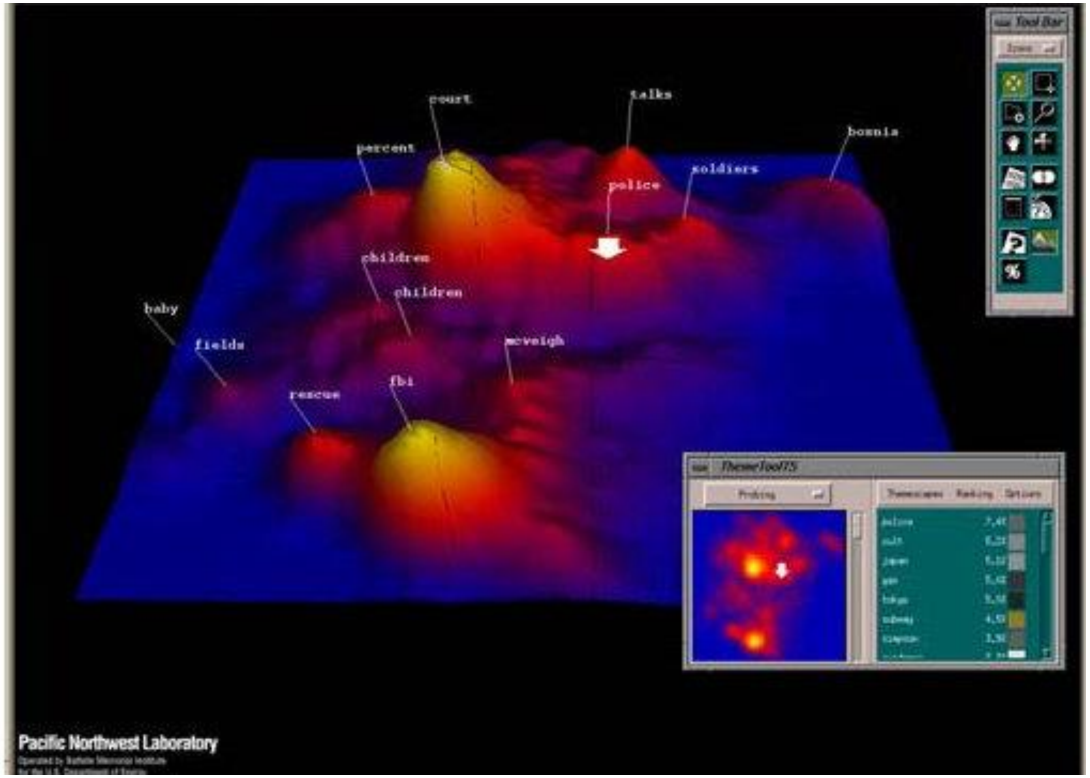


'Poverty map' based on 39 indicators from World Bank statistics (1992)

SOM Example: ThemeScape

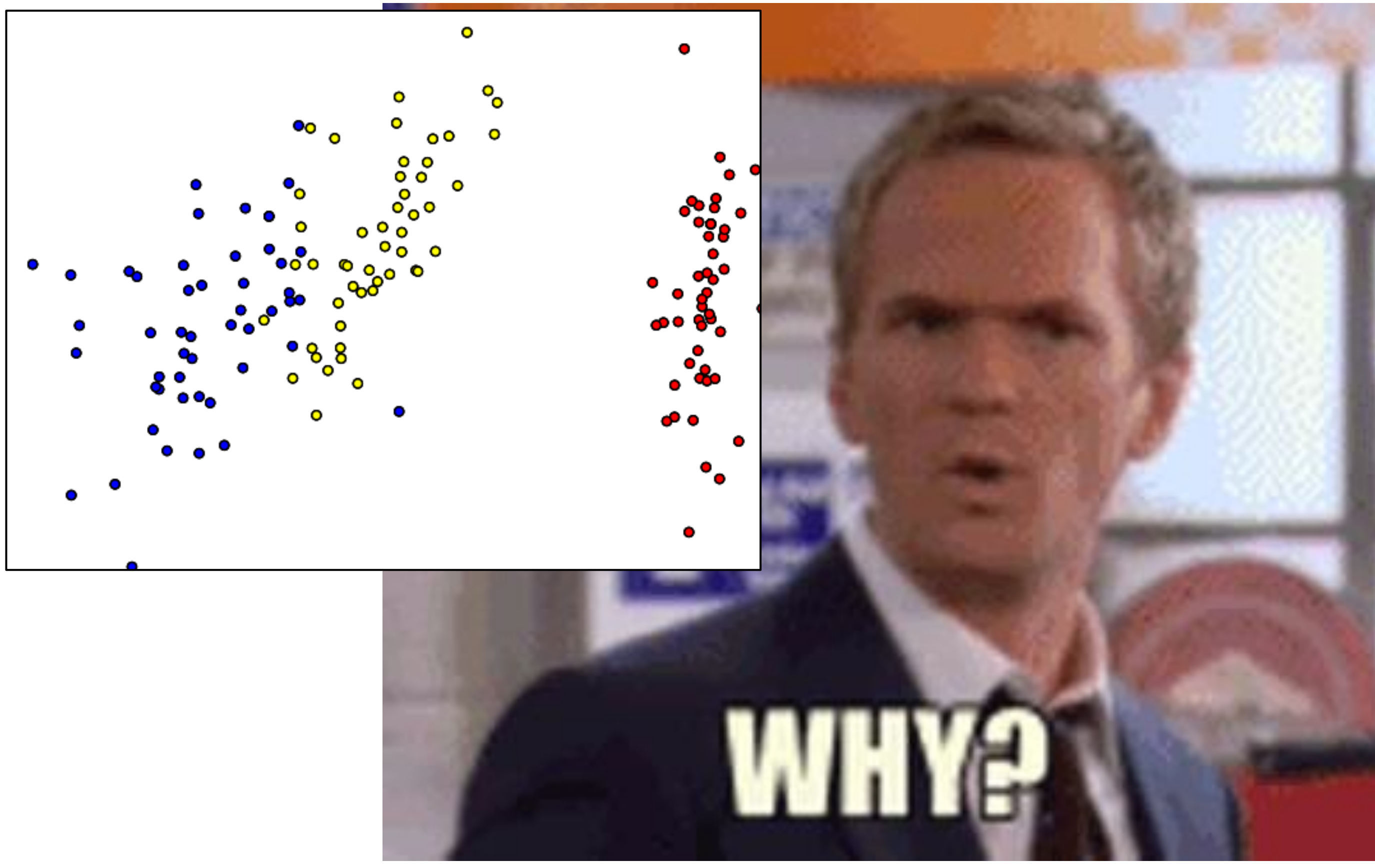
Height represents density or number of documents in the region

Invented at Pacific Northwest National Lab (PNNL)



But….

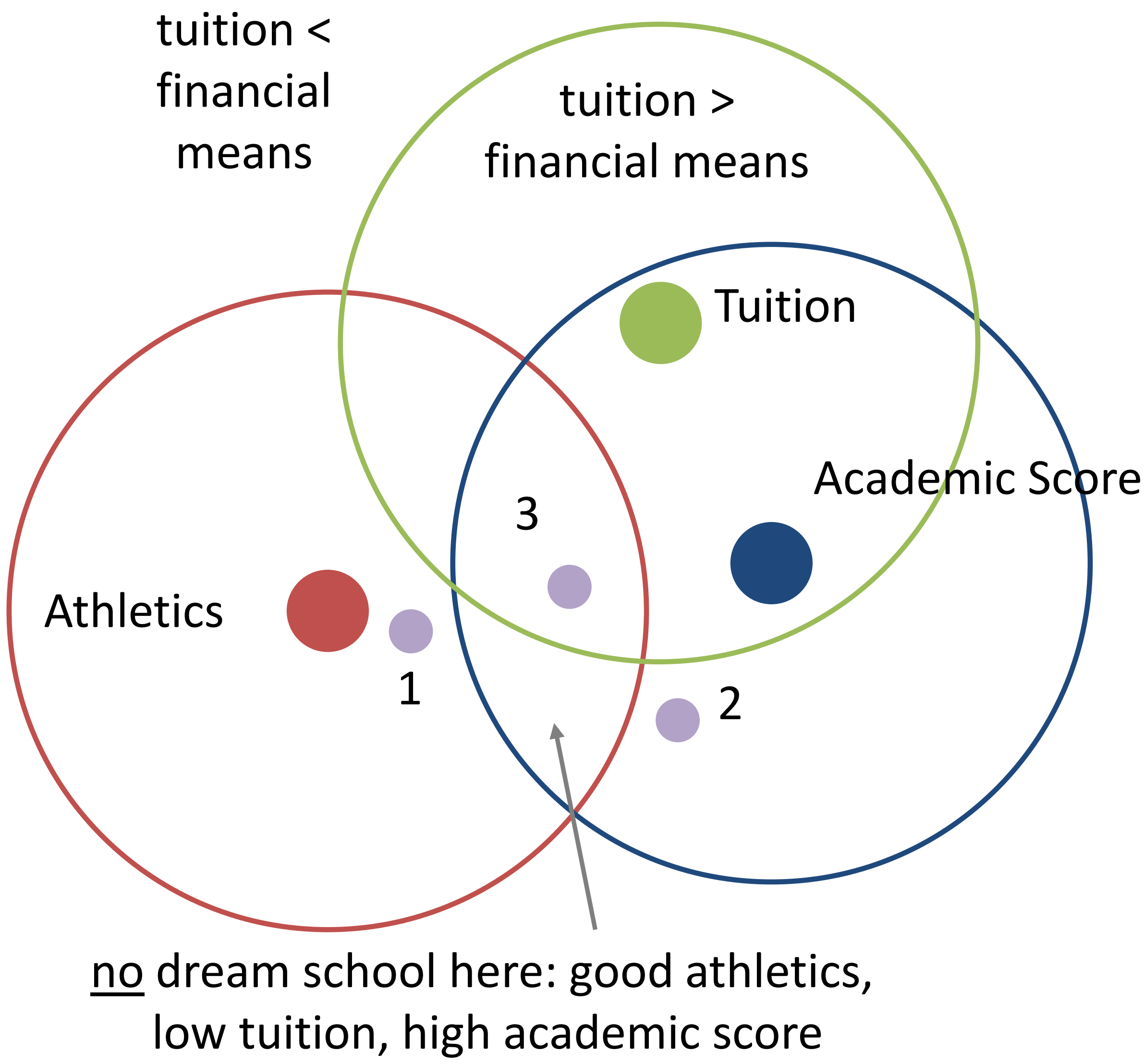
- Japanese cars
- European cars
- US cars



…ARE THESE CLUSTERS SO DIFFERENT?

We Need to Map The Attributes, Too

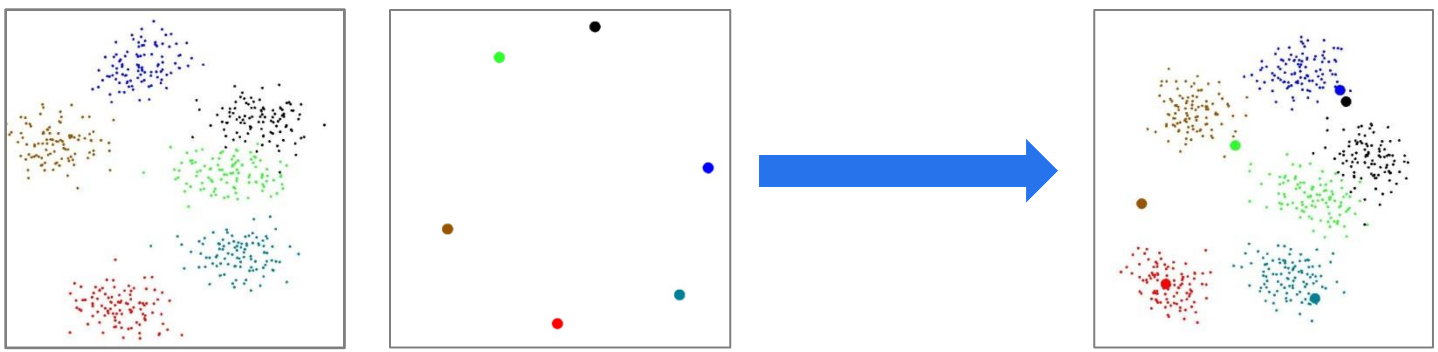
Example College Selection



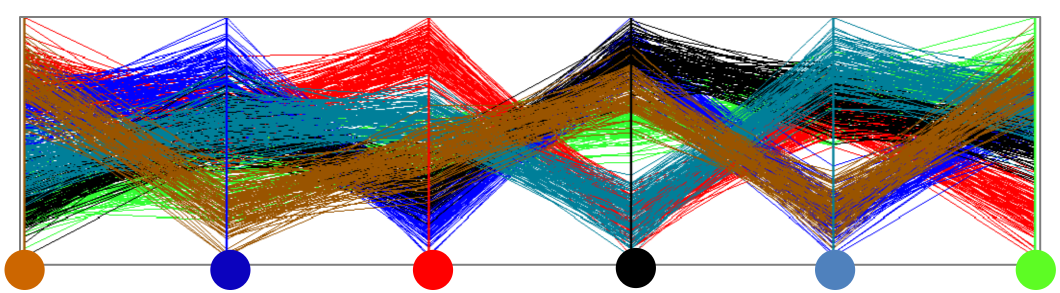
The Data Context Map

Best of both worlds

- similarity layout of the data based on vector similarity
- similarity layout of the attributes based on pairwise correlation

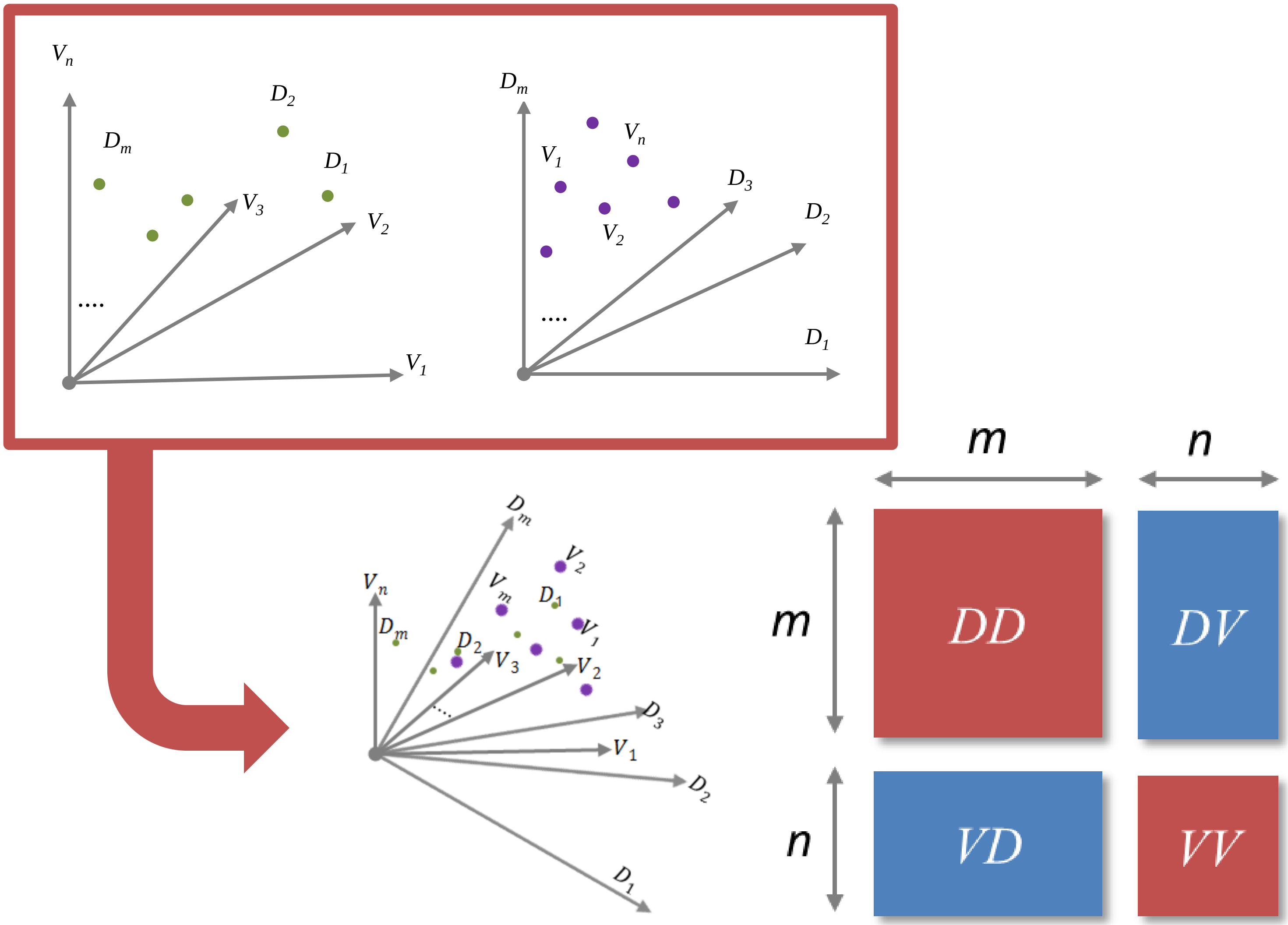


data attributes data + attributes

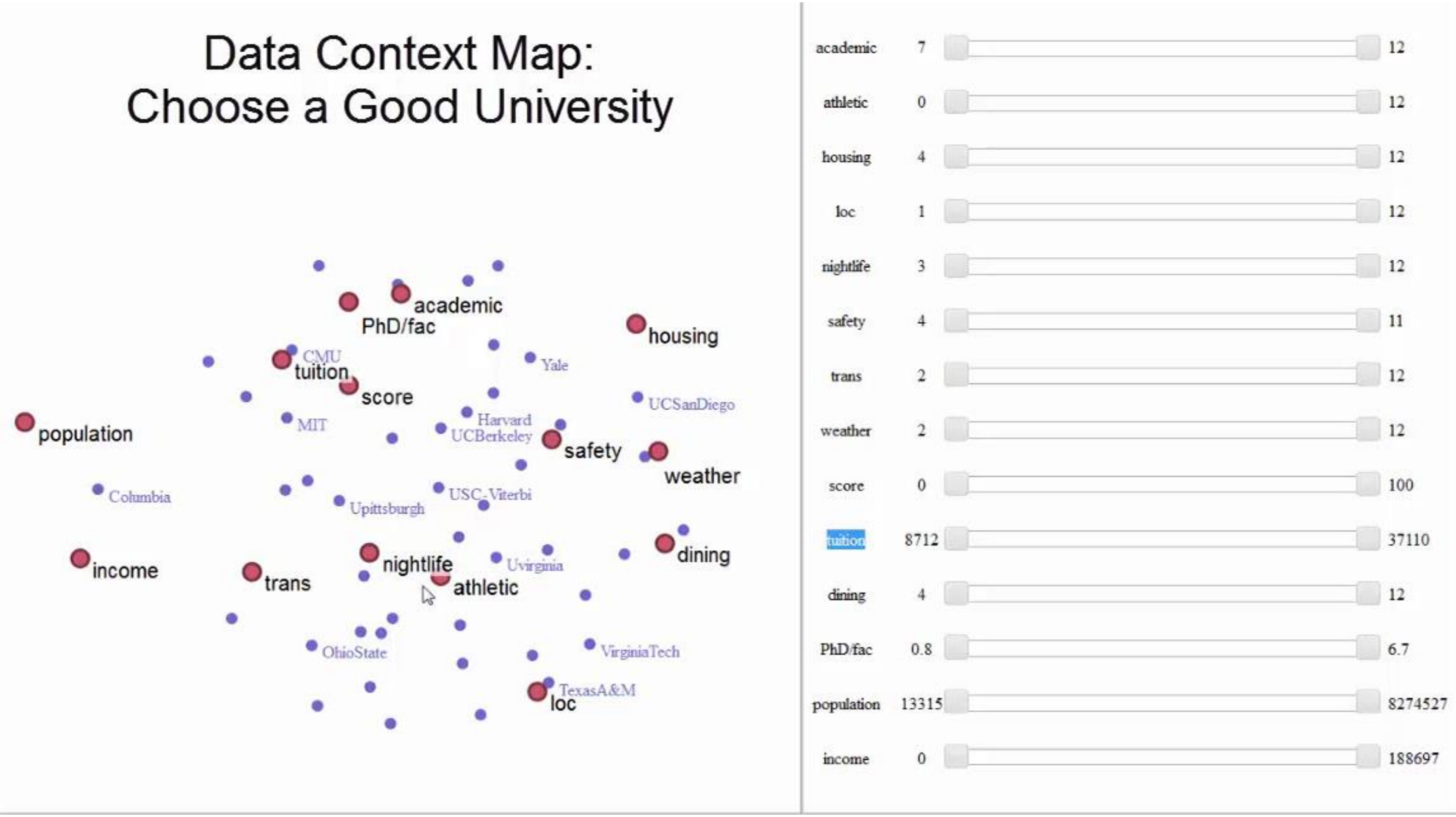


[Cheng and Mueller, TVCG 2016]

Achieved by Joint Matrix Optimization



The Data Context Map



What About Categorical Variables?

You will need to use correspondence analysis (CA)

- CA is PCA for categorical variables
- related to factor analysis

Correspondence Analysis (CA)

more info

Example:

	Smoking Category				
Staff Group	(1) None	(2) Light	(3) Medium	(4) Heavy	Row Totals
(1) Senior Managers	4	2	3	2	11
(2) Junior Managers	4	3	7	4	18
(3) Senior Employees	25	10	12	4	51
(4) Junior Employees	18	24	33	13	88
(5) Secretaries	10	6	7	2	25
Column Totals	61	45	62	25	193

There are two high-D spaces

- 4D (column) space spanned by smoking habits – plot staff group
- 5D (row) space spanned by staff group – plot smoking habits

Are these two spaces (the rows and columns) independent ?

- this occurs when the χ^2 statistics of the table is insignificant

CA Eigen Analysis

Staff Group	Smoking Category (1) None	(2) Light	(3) Medium	(4) Heavy	Row Totals
(1) Senior Managers	4	2	3	2	11
(2) Junior Managers	4	3	7	4	18
(3) Senior Employees	25	10	12	4	51
(4) Junior Employees	18	24	33	13	88
(5) Secretaries	10	6	7	2	25
Column Totals	61	45	62	25	193

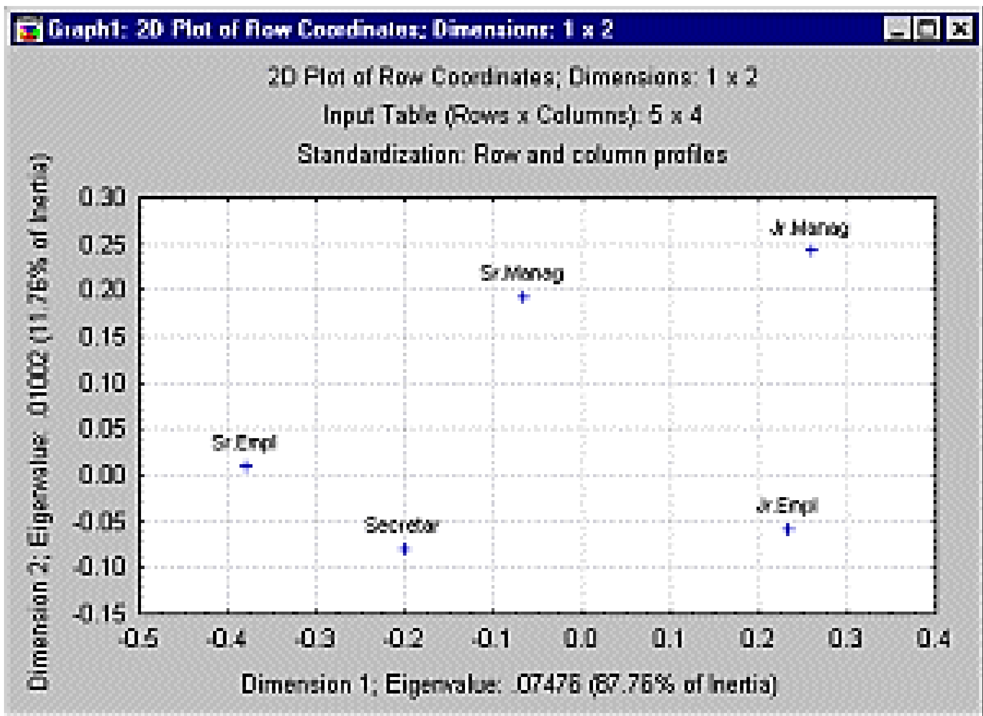
x

Smoking Category	(1) Senior Managers	(2) Junior Managers	(3) Senior Employees	(4) Junior Employees	(5) Secretaries	Column Totals
(1) None	4	4	25	18	10	61
(2) Light	2	3	10	24	6	45
(3) Medium	3	7	12	33	7	62
(4) Heavy	2	4	4	13	2	25
Row Totals	11	18	51	88	25	193

--> distance matrix of employees

Let's do some plotting

- compute distance matrix of the rows CC^T
- compute Eigenvector matrix **U** and the Eigenvalue matrix **D**
- sort eigenvectors by values, pick two major vectors, create 2D plot



-- senior employees most similar to secretaries

Eigenvalues and Inertia for all Dimensions
Input Table (Rows x Columns): 5 x 4
Total Inertia = .08519 Chi² = 16.442

No. of Dims	Singular Values	Eigen-Values	Perc. of Inertia	Cumulatv Percent	Chi Squares
1	.273421	.074759	87.75587	87.7559	14.42851
2	.100086	.010017	11.75865	99.5145	1.93332
3	.020337	.000414	.48547	100.0000	.07982

CA Eigen Analysis

Staff Group	Smoking Category (1) None	(2) Light	(3) Medium	(4) Heavy	Row Totals
(1) Senior Managers	4	2	3	2	11
(2) Junior Managers	4	3	7	4	18
(3) Senior Employees	25	10	12	4	51
(4) Junior Employees	18	24	33	13	88
(5) Secretaries	10	6	7	2	25
Column Totals	61	45	62	25	193

Next:

- compute distance matrix of the columns $\mathbf{C}^T\mathbf{C}$
- compute Eigenvector matrix **V** (gives the same Eigenvalue matrix **D**)
- sort eigenvectors by value
- pick two major vectors
- create 2D plot of smoking categories

Smoking Category	(1) Senior Managers	(2) Junior Managers	(3) Senior Employees	(4) Junior Employees	(5) Secretaries	Column Totals
(1) None	4	4	25	18	10	61
(2) Light	2	3	10	24	6	45
(3) Medium	3	7	12	33	7	62
(4) Heavy	2	4	4	13	2	25
Row Totals	11	18	51	88	25	193

x

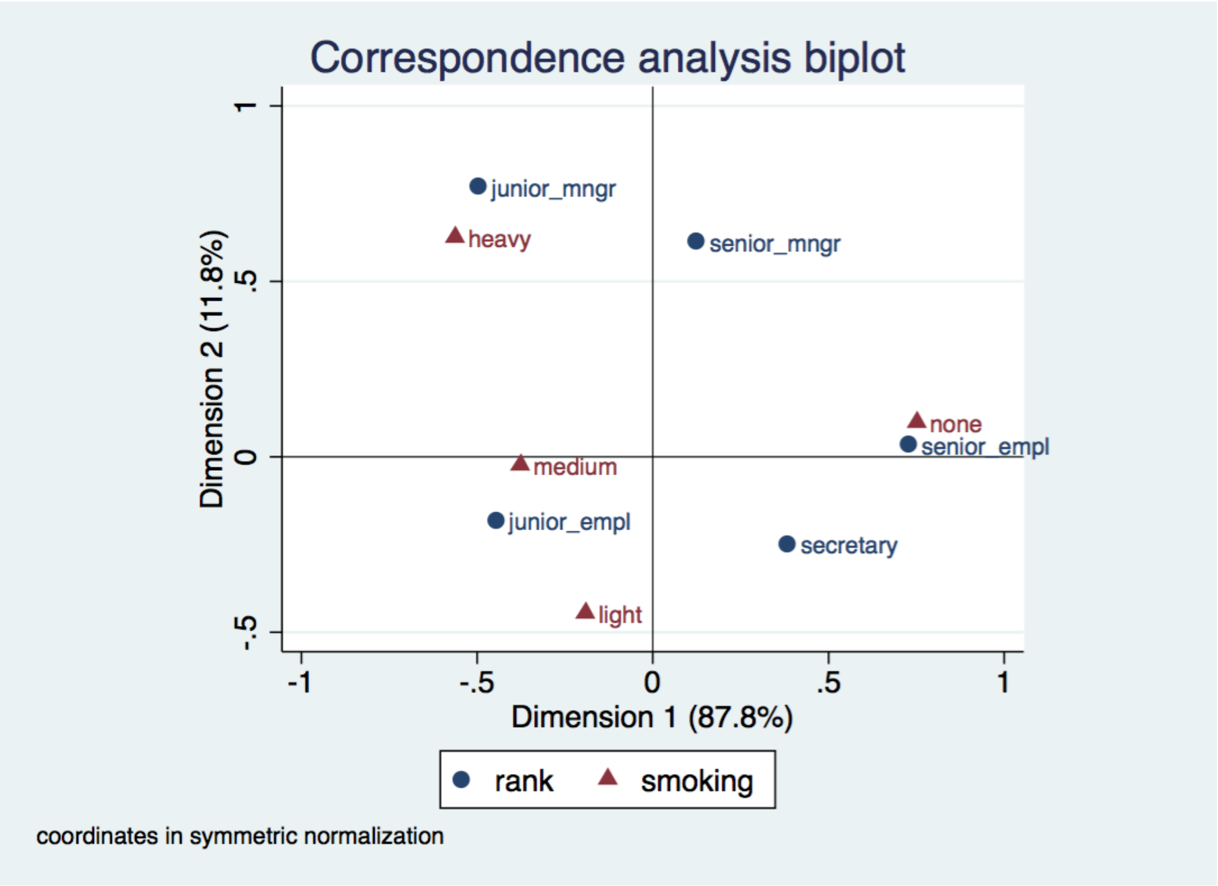
Staff Group	Smoking Category (1) None	(2) Light	(3) Medium	(4) Heavy	Row Totals
(1) Senior Managers	4	2	3	2	11
(2) Junior Managers	4	3	7	4	18
(3) Senior Employees	25	10	12	4	51
(4) Junior Employees	18	24	33	13	88
(5) Secretaries	10	6	7	2	25
Column Totals	61	45	62	25	193

--> distance matrix of smoking habits

Following (next slide):

- combine the plots of **U** and **V**
- if the χ^2 statistics was significant we should see some dependencies

Combined CA Plot



Interpretation sample (using the χ^2 frequentist mindset)

- *relatively speaking*, there are more non-smoking senior employees

Extending to Cases

Case Number	Senior Manager	Junior Manager	Senior Employee	Junior Employee	Secretary	None	Light	Medium	Heavy
1	1	0	0	0	0	1	0	0	0
2	1	0	0	0	0	1	0	0	0
3	1	0	0	0	0	1	0	0	0
4	1	0	0	0	0	1	0	0	0
5	1	0	0	0	0	0	1	0	0
...	.	.	.	.	.	.	.	.	.
...	.	.	.	.	.	.	.	.	.
...	.	.	.	.	.	.	.	.	.
191	0	0	0	0	1	0	0	1	0
192	0	0	0	0	1	0	0	0	1
193	0	0	0	0	1	0	0	0	1

Plot would now show 193 cases and 9 variables

Multiple Correspondence Analysis

Extension where there are more than 2 categorical variables

	SURVIVAL		AGE			LOCATION		
Case No.	NO	YES	LESST50	A50TO69	OVER69	TOKYO	BOSTON	GLAMORGN
1	0	1	0	1	0	0	0	1
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	0	1	0	0	1	0	0	1
...	.	.	.	.	.	.	.	.
...	.	.	.	.	.	.	.	.
...	.	.	.	.	.	.	.	.
762	1	0	0	1	0	1	0	0
763	0	1	1	0	0	0	1	0
764	0	1	0	1	0	0	0	1

Let’s call it matrix X

Multiple Correspondence Analysis

Compute X'X to get the Burt Table

	SURVIVAL		AGE			LOCATION		
	NO	YES	<50	50-69	69+	TOKYO	BOSTON	GLAMORGN
SURVIVAL:NO	210	0	68	93	49	60	82	68
SURVIVAL:YES	0	554	212	258	84	230	171	153
AGE:UNDER_50	68	212	280	0	0	151	58	71
AGE:A_50TO69	93	258	0	351	0	120	122	109
AGE:OVER_69	49	84	0	0	133	19	73	41
LOCATION:TOKYO	60	230	151	120	19	290	0	0
LOCATION:BOSTON	82	171	58	122	73	0	253	0
LOCATION:GLAMORGN	68	153	71	109	41	0	0	221

Compute Eigenvectors and Eigenvalues

- keep top two Eigenvectors/values
- visualize the attribute loadings of these two Eigenvectors into the Burt table plot (the loadings are the coordinates)

Larger MCA Example

Results of a survey of car owners and car attributes

Burt Table

	American	European	Japanese	Large	Medium	Small	Family	Sporty	Work	1 Income	2 Incomes	Own	Rent	Married	Married with Kids	Single	Single with Kids	Female	Male
American	125	0	0	36	60	29	81	24	20	58	67	93	32	37	50	32	6	58	67
European	0	44	0	4	20	20	17	23	4	18	26	38	6	13	15	15	1	21	23
Japanese	0	0	165	2	61	102	76	59	30	74	91	111	54	51	44	62	8	70	95
Large	36	4	2	42	0	0	30	1	11	20	22	35	7	9	21	11	1	17	25
Medium	60	20	61	0	141	0	89	39	13	57	84	106	35	42	51	40	8	70	71
Small	29	20	102	0	0	151	55	66	30	73	78	101	50	50	37	58	6	62	89
Family	81	17	76	30	89	55	174	0	0	69	105	130	44	50	79	35	10	83	91
Sporty	24	23	59	1	39	66	0	106	0	55	51	71	35	35	12	57	2	44	62
Work	20	4	30	11	13	30	0	0	54	26	28	41	13	16	18	17	3	22	32
1 Income	58	18	74	20	57	73	69	55	26	150	0	80	70	10	27	99	14	47	103
2 Incomes	67	26	91	22	84	78	105	51	28	0	184	162	22	91	82	10	1	102	82
Own	93	38	111	35	106	101	130	71	41	80	162	242	0	76	106	52	8	114	128
Rent	32	6	54	7	35	50	44	35	13	70	22	0	92	25	3	57	7	35	57
Married	37	13	51	9	42	50	50	35	16	10	91	76	25	101	0	0	0	53	48
Married with Kids	50	15	44	21	51	37	79	12	18	27	82	106	3	0	109	0	0	48	61
Single	32	15	62	11	40	58	35	57	17	99	10	52	57	0	0	109	0	35	74
Single with Kids	6	1	8	1	8	6	10	2	3	14	1	8	7	0	0	0	15	13	2
Female	58	21	70	17	70	62	83	44	22	47	102	114	35	53	48	35	13	149	0
Male	67	23	95	25	71	89	91	62	32	103	82	128	57	48	61	74	2	0	185

more info see here

MCA Example (2)

Summary table:

Inertia and Chi-Square Decomposition					
Singular Value	Principal Inertia	Chi-Square	Percent	Cumulative Percent	4 8 12 16 20 ----+----+----+----+----+---
0.56934	0.32415	970.77	18.91	18.91	************************
0.48352	0.23380	700.17	13.64	32.55	*****************
0.42716	0.18247	546.45	10.64	43.19	*************
0.41215	0.16987	508.73	9.91	53.10	************
0.38773	0.15033	450.22	8.77	61.87	***********
0.38520	0.14838	444.35	8.66	70.52	***********
0.34066	0.11605	347.55	6.77	77.29	********
0.32983	0.10879	325.79	6.35	83.64	********
0.31517	0.09933	297.47	5.79	89.43	*******
0.28069	0.07879	235.95	4.60	94.03	******
0.26115	0.06820	204.24	3.98	98.01	*****
0.18477	0.03414	102.24	1.99	100.00	**
Total	1.71429	5133.92	100.00		

Degrees of Freedom = 324

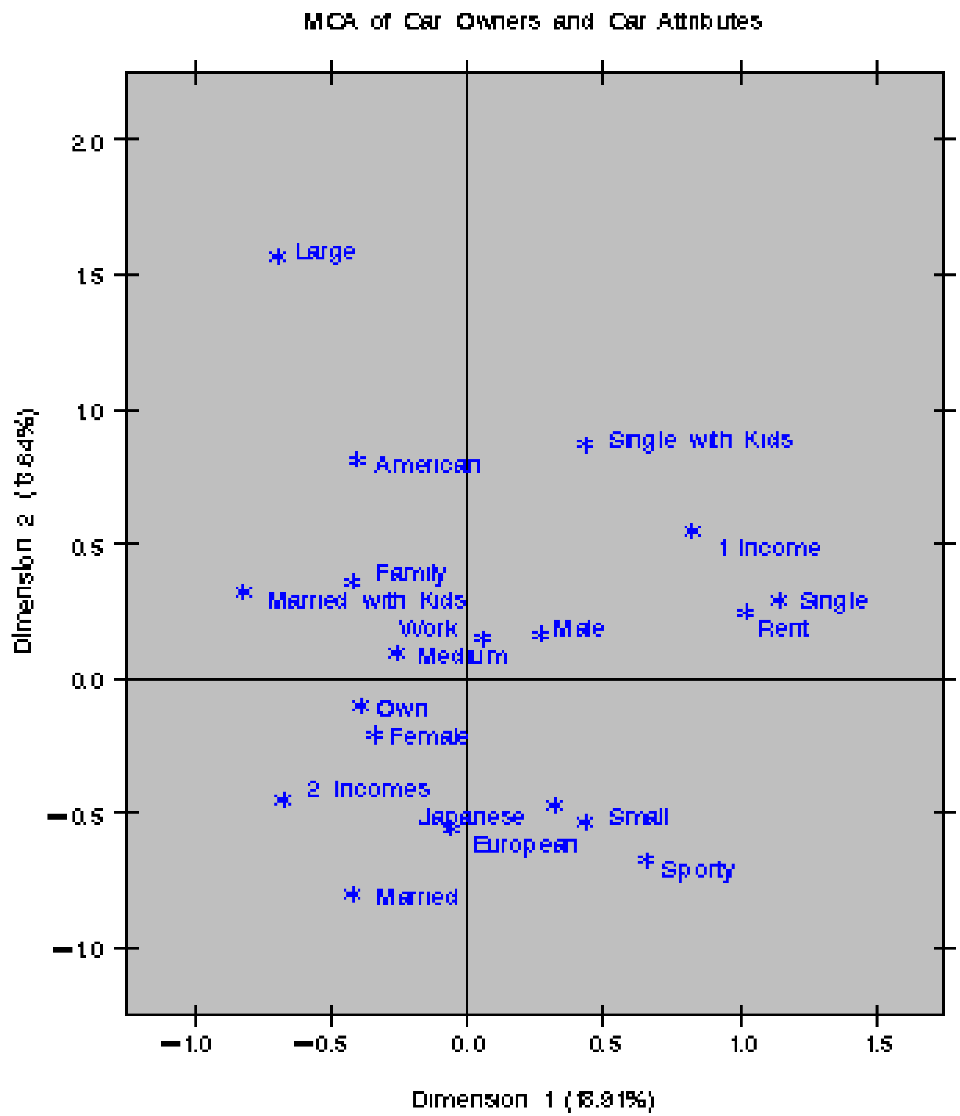
MCA EXAMPLE (3)

Most influential column points (loadings):

Column Coordinates		
	Dim1	Dim2
American	-0.4035	0.8129
European	-0.0568	-0.5552
Japanese	0.3208	-0.4678
Large	-0.6949	1.5666
Medium	-0.2562	0.0965
Small	0.4326	-0.5258
Family	-0.4201	0.3602
Sporty	0.6604	-0.6696
Work	0.0575	0.1539
1 Income	0.8251	0.5472
2 Incomes	-0.6727	-0.4461
Own	-0.3887	-0.0943
Rent	1.0225	0.2480
Married	-0.4169	-0.7954
Married with Kids	-0.8200	0.3237
Single	1.1461	0.2930
Single with Kids	0.4373	0.8736
Female	-0.3365	-0.2057
Male	0.2710	0.1656

MCA Example (4)

Burt table plot:



Plot Observations

Top-right quadrant:

- categories single, single with kids, 1 income, and renting a home are associated

Proceeding clockwise:

- the categories sporty, small, and Japanese are associated
- being married, owning your own home, and having two incomes are associated
- having children is associated with owning a large American family car

Such information could be used in market research to identify target audiences for advertisements

Gartner Magic Quadrant

A Gartner Magic Quadrant is a culmination of research in a specific market, providing a wide-angle view of the relative positions of the market's competitors

This concept can be used for other dimension pairs as well

- essentially require to think of a segmentation of the 4 quadrants

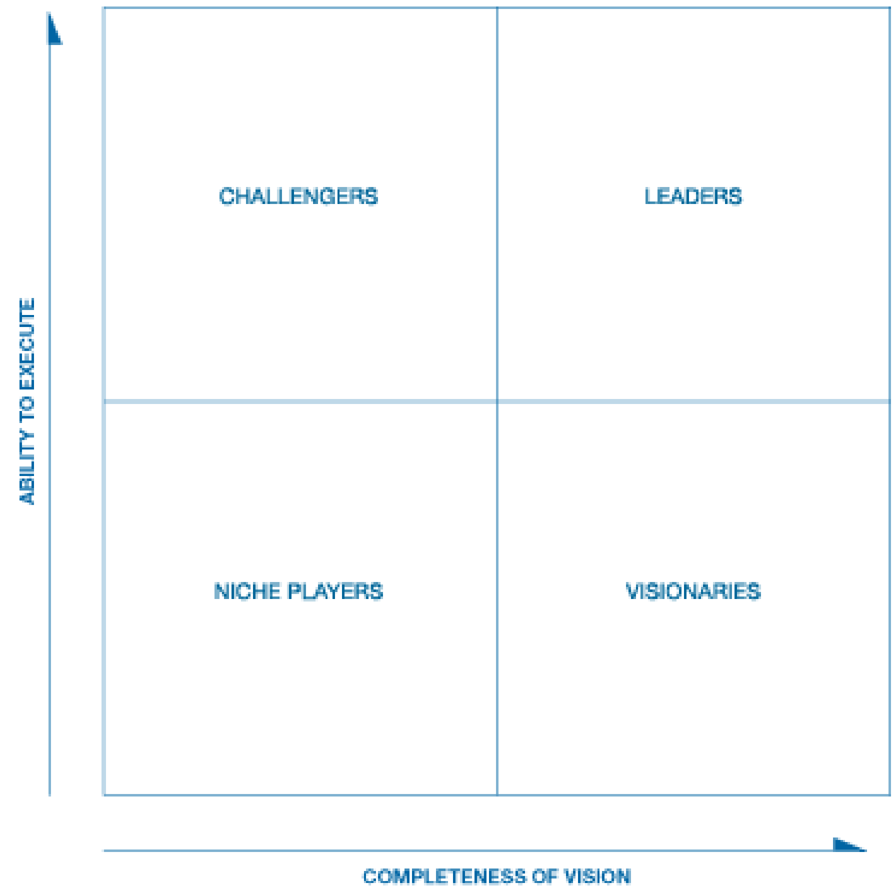
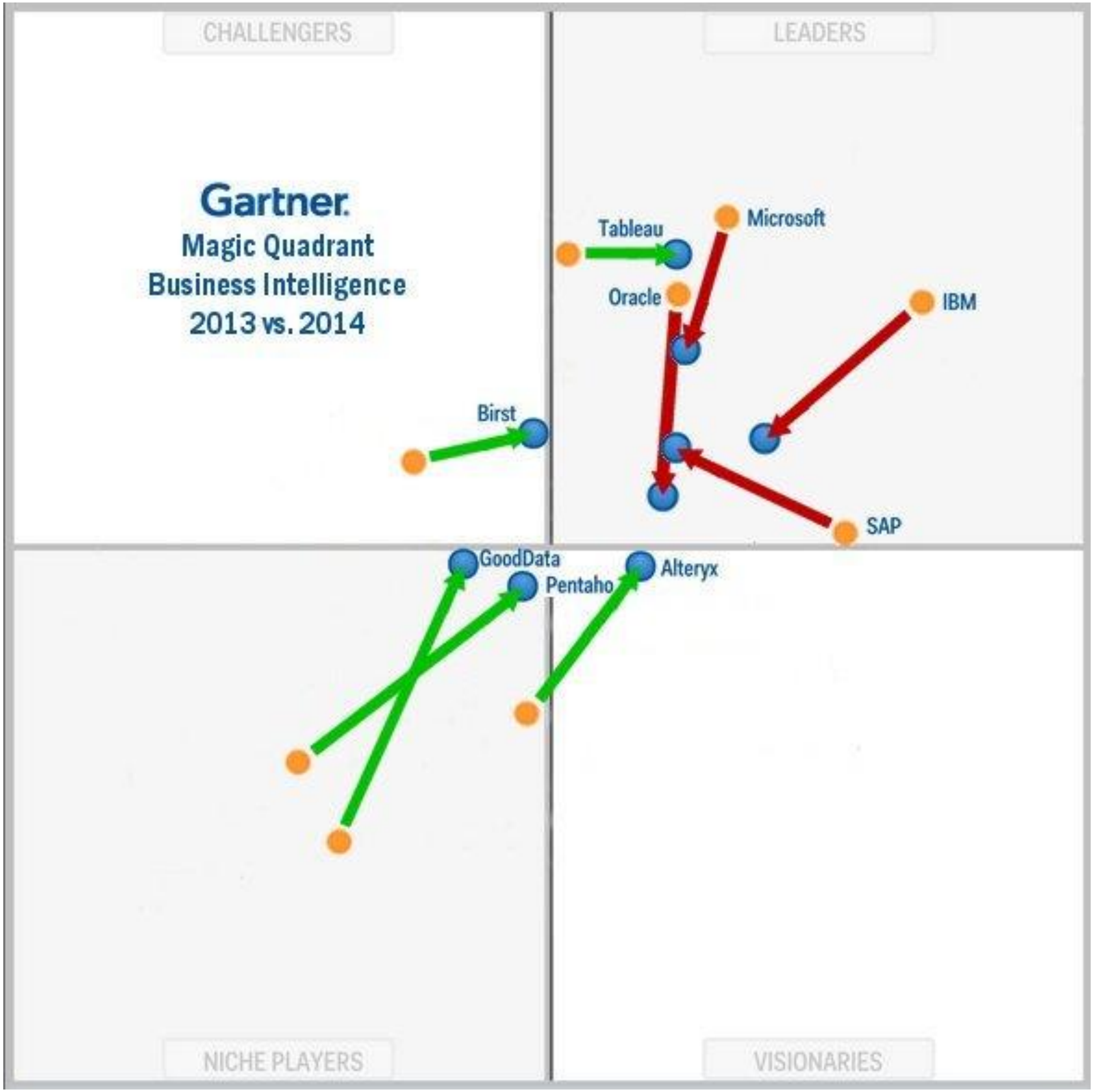


Figure 1. Magic Quadrant for Business Intelligence and Analytics Platforms

Source: Gartner (February 2014)


CHALLENGERS
LEADERS
Gartner
Magic Quadrant
Business Intelligence
2013 vs. 2014
Tableau
Microsoft
Oracle
IBM
Birst
SAP
GoodData
Pentaho
Alteryx
NICHE PLAYERS
VISIONARIES